Solutions Key
Triangle Congruence

ARE YOU READY? PAGE 213

1. F **2.** D

3. B **4.** A

5. E **6.** 35°

7. 90°

8–11. Check students' drawings.

12.
$$\begin{aligned}\tfrac{9}{2}x + 7 &= 25\\ -7 \quad & \;\; -7\\ \tfrac{9}{2}x &= 18\\ x &= \tfrac{2(18)}{9} = 4\end{aligned}$$

13.
$$\begin{aligned}3x - \tfrac{2}{3} &= \tfrac{4}{3}\\ +\tfrac{2}{3} \quad & \;\; +\tfrac{2}{3}\\ 3x &= 2\\ x &= \tfrac{2}{3}\end{aligned}$$

14.
$$\begin{aligned}x - \tfrac{1}{5} &= \tfrac{12}{5}\\ +\tfrac{1}{5} \quad & \;\; +\tfrac{1}{5}\\ x &= \tfrac{13}{5} = 2\tfrac{3}{5}\end{aligned}$$

15.
$$\begin{aligned}2y &= 5y - \tfrac{21}{2}\\ -5y \quad & \;\; -5y\\ -3y &= -\tfrac{21}{2}\\ y &= \tfrac{7}{2} = 3\tfrac{1}{2}\end{aligned}$$

16. t is 3 times m.
$t = 3m$

17. Twice x is 9 ft.
$2x = 9$

18. 53° + twice y is 90°.
$53 + 2y = 90$

19. Price r is price p less 25.
$r = p - 25$

20. Half j is b plus 5 oz.
$\frac{1}{2}j = b + 5$

4-1 CLASSIFYING TRIANGLES, PAGES 216–221

CHECK IT OUT!

1. $\angle FHG$ and $\angle EHF$ are complementary.
$$\begin{aligned}m\angle FHG + m\angle EHF &= 90°\\ m\angle FHG + 30° &= 90°\\ m\angle FHG &= 60°\end{aligned}$$
All ∠s are equal. So $\triangle FHG$ is equiangular by definition.

2. $AC = AB = 15$
No sides are congruent. So $\triangle ACD$ is scalene.

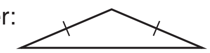
3. Step 1 Find the value of y.
$$\begin{aligned}\overline{FG} &\cong \overline{GH}\\ FG &= GH\\ 3y - 4 &= 2y + 3\\ 3y &= 2y + 7\\ y &= 7\end{aligned}$$
Step 2 Substitute 7 for y.
$$\begin{aligned}FG &= 3y - 4 = 3(7) - 4 = 17\\ GH &= 2y + 3 = 2(7) + 3 = 17\\ FH &= 5y - 18 = 5(7) - 18 = 17\end{aligned}$$

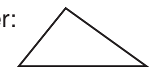
4a. $P = 3(7) = 21$ in.
$100 \div 21 = 4\frac{16}{21}$
4 triangles

b. $P = 3(10) = 30$ in.
$100 \div 30 = 3\frac{1}{3}$
3 triangles

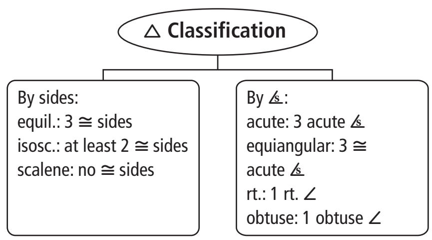
THINK AND DISCUSS

1. $\overline{DE}$, $\overline{EF}$, $\angle E$; $\overline{EF}$, $\overline{FD}$, $\angle F$; $\overline{FD}$, $\overline{DE}$, $\angle D$

2. Possible answer: 

3. No; all 3 ∠s in an acute △ must be acute, but they do not have to have the same measure; possible answer: 

4. In an equil. rt. △, all 3 sides have the same length. By the Pyth. Thm., the 3 side lengths are related by the formula $c^2 = a^2 + b^2$, making the hyp. c greater than either a or b. So the 3 sides cannot have the same length.

5.


EXERCISES

GUIDED PRACTICE

1. An equilateral triangle has three congruent sides.

2. One angle is obtuse and the other two angles are acute.

3. $\angle DBC$ is a rt. ∠.
So $\triangle DBC$ is a rt. △.

4. $\angle ABD$ and $\angle DBC$ are supp.
$$\begin{aligned}\angle ABD + \angle DBC &= 180°\\ \angle ABD + 90 &= 180\\ \angle ABD &= 90°\end{aligned}$$
$\angle ABD$ is a rt. ∠. So $\triangle ABD$ is a rt. △.

5.
$$\begin{aligned}m\angle ADC &= m\angle ADB + m\angle BDC\\ &= 31 + 70 = 101°\end{aligned}$$
$\angle ADC$ is obtuse. So $\triangle ADC$ is an obtuse △.

6. $EG = 3 + 3 = 6$,
$EH = 8$, $GH = 8$
$\overline{EH} \cong \overline{GH}$
Exactly two sides are $\cong$, so $\triangle EGH$ is isosc.

7. $EF = 3$, $EH = 8$,
$FH = 7.4$
No sides are congruent, so $\triangle EFH$ is scalene.

8. $GF = 3$, $GH = 8$, $FH = 7.4$
No sides are congruent, so $\triangle HFG$ is scalene.

9. **Step 1** Find y.
$6y = 4y + 12$
$2y = 12$
$y = 6$
Step 2 Find side lengths.
$\triangle$ is equilateral, so all three side lengths $= 6y = 36$.

10. **Step 1** Find x.
$2x + 1.7 = x + 2.4$
$2x = x + 0.7$
$x = 0.7$
Step 2 Find side lengths.
$x + 2.4 = 0.7 + 2.4 = 3.1$
$2x + 1.7 = 2(0.7) + 1.7 = 1.4 + 1.7 = 3.1$
$4x + 0.5 = 4(0.7) + 0.5 = 2.8 + 0.5 = 3.3$

11. Perimeter is
$P = 3 + 3 + 1.5$
$= 7.5$ cm
$50 \div 7.5 = 6\frac{2}{3}$ earrings
The jeweler can make 6 earrings.

PRACTICE AND PROBLEM SOLVING

12. $m\angle BEA = 90°$; rt. $\triangle$

13. $m\angle BCD = 60 + 60 = 120°$; obtuse

14. $m\angle ABC = 30 + 30 = 60°$
$m\angle ABC = m\angle ACB = m\angle BAC$; equiangular

15. $\overline{PS} \cong \overline{ST} \cong \overline{PT}$; equilateral

16. $\overline{PS} \cong \overline{RS}$, *so* $PS = RS = 10$; $RP = 17$; isosc.

17. $RT = 10 + 10 = 20$, $RP = 17$, $PT = 10$; scalene

18. **Step 1** Find z.
$3z - 1 = z + 5$
$3z = z + 6$
$2z = 6$
$z = 3$
Step 2 Find side lengths.
$z + 5 = 3 + 5 = 8$
$3z - 1 = 3(3) - 1 = 8$
$4z - 4 = 4(3) - 4 = 8$

19. **Step 1** Find x.
$8x + 1.4 = 2x + 6.8$
$8x = 2x + 5.4$
$6x = 5.4$
$x = 0.9$
Step 2 Find side lengths.
$8x + 1.4 = 8(0.9) + 1.4$
$= 7.2 + 1.4$
$= 8.6$
$2x + 6.8 = 2(0.9) + 6.8$
$= 1.8 + 6.8$
$= 8.6$

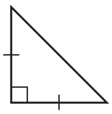
20a. Check students' drawings.
$\overline{XY}$, $\overline{YZ}$, $\overline{XZ}$, $\angle X$, $\angle Y$, $\angle Z$

b. Possible answer: scalene obtuse

21. $PQ + PR + QR = 60$
$PQ + PQ + \frac{4}{3}PQ = 60$
$\frac{10}{3}PQ = 60$
$PQ = \frac{3}{10}(60) = 18$ ft
$PR = PQ = 18$ ft
$QR = \frac{4}{3}PQ = \frac{4}{3}(18) = 24$ ft

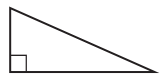
22. $150 \div 60 = 2\frac{1}{2}$; 2 complete trusses

23.

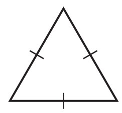


24. Not possible: an equiangular $\triangle$ has only acute $\angle$s.

25.



26.



27. Not possible: an equiangular $\triangle$ must also be equilateral.

28.

29. Let x represent each side length.
$x + x + x = 105$
$3x = 105$
$x = 35$ in.

30. $\overline{AB} \cong \overline{AC}$, so $\triangle$ is isosc.
$\angle BAC$ and $\angle CAD$ are supp., and $\angle CAD$ is acute; so $\angle BAC$ is obtuse.
$\triangle ABC$ is isosc. obtuse.

31. $\overline{AC} \cong \overline{CD}$ and $m\angle ACD = 90°$.
$\triangle ACD$ is isosc. rt.

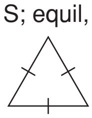
32. $(4x - 1) + (4x - 1) + x = 34$
$9x - 2 = 34$
$9x = 36$
$x = 4$

33a. E 22nd Street side $= \frac{1}{2}$(Broadway side) $- 8$
$= \frac{1}{2}(190) - 8 = 87$ ft
5th Avenue side $= 2$(E 22nd Street side) $- 1$
$= 2(87) - 1$
$= 173$ ft

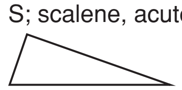
b. All sides are different, so $\triangle$ is scalene.

34. No; yes; not every isosc. $\triangle$ is equil. because only 2 of the 3 sides must be $\cong$. Every equil. $\triangle$ has 3 $\cong$ sides, and the def. of an isosc. $\triangle$ requires that at least 2 sides be $\cong$.

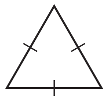
35. S; equil, acute



36. S; scalene, acute



37. A; 3 congruent sides, so always satisfies isosceles $\triangle$ classification



38. $s = \frac{P}{3}$. The perimeter of an equil. $\triangle$ is 3 times the length of any 1 side, or $P = 3s$. Solve this formula for s by dividing both sides by 3.

39. Check students' constructions.

40a.
$$DE^2 = AD^2 + AE^2$$
$$= 5^2 + \left(\frac{10}{2}\right)^2$$
$$= 25 + 25 = 50$$
$DE = \sqrt{50} = 5\sqrt{2}$ cm

Think: $\overline{CE} \cong \overline{DE}$.

$CE = DE = 5\sqrt{2}$ cm

b. Think: DE bisects $\angle AEF$.
$$m\angle DEF = \frac{1}{2} m\angle AEF$$
$$= \frac{1}{2}(90) = 45°$$
Think: $\angle CEF \cong \angle DEF$, so $m\angle CEF = 45°$.
$$m\angle DEC = m\angle DEF + m\angle CEF$$
$$= 45 + 45 = 90°$$

c. $CE = DE$ and $m\angle DEC = 90°$

isosc. $\triangle$; rt. $\triangle$

TEST PREP

41. D
$$3s = P$$
$$3s = 36\frac{2}{3}$$
$$s = \frac{1}{3}\left(36 + \frac{2}{3}\right)$$
$$= 12\frac{2}{9} \text{ in.}$$

42. F

By graphing, $RT \cong RS \not\cong ST$, so $\triangle RST$ is isosc.

43. D

$\triangle LMN$ has no rt. $\angle$.

44. 3
$$P = AB + BC + AC$$
$$= \frac{1}{2}x + \frac{1}{4} + \frac{5}{2} - x + \frac{1}{2}x + \frac{1}{4}$$
$$= \left(\frac{1}{2} - 1 + \frac{1}{2}\right)x + \frac{1}{4} + \frac{5}{2} + \frac{1}{4}$$
$$= 3$$

CHALLENGE AND EXTEND

45. It is an isosc. $\triangle$ since 2 sides of the $\triangle$ have length a. It is also a rt. $\triangle$ since 2 sides of the $\triangle$ lie on the coord. axes and form a rt. $\angle$.

46.

Statements	Reasons
1. $\triangle ABC$ is equiangular.	1. Given
2. $\angle A \cong \angle B \cong \angle C$	2. Def. of equiangular $\triangle$
3. $\overline{EF} \parallel \overline{AC}$	3. Given
4. $\angle BEF \cong \angle A$, $\angle BFE \cong \angle C$	4. Corr. $\angle$s Post.
5. $\angle BEF \cong \angle B$, $\angle BFE \cong \angle B$	5. Trans. Prop. of $\cong$
6. $\angle BEF \cong \angle BFE$	6. $\angle$s $\cong$ to the same $\angle$ are $\cong$.
7. $\triangle EFB$ is equiangular.	7. Def. of equiangular $\triangle$

47. Think: Each side has the same measure. Use the expression $y + 10$ for this measure.
$$3(y + 10) = 21$$
$$3y + 30 = 21$$
$$3y = -9$$
$$y = -3$$

48. **Step 1** Find x. Think: Average of $x + 12$, $3x + 4$, and $8x - 16$ is 24.
$$\frac{1}{3}(x + 12 + 3x + 4 + 8x - 16) = 24$$
$$\frac{1}{3}(12x) = 24$$
$$4x = 24$$
$$x = 6$$
Step 2 Find side lengths.

$x + 12 = 6 + 12 = 18$

$3x + 4 = 3(6) + 4 = 22$

$8x - 16 = 8(6) - 16 = 32$

longest side $-$ average $= 32 - 24 = 8$

SPIRAL REVIEW

49. $y = x^2$

50. $y = x$

51. $y = x^2$

52. F; skew lines do not intersect and are not parallel.

53. T

54. F; Possible answer: 30 has a 0 in the ones place, but 30 is not a multiple of 20.

55. $y = 4x + 2$ has slope 4. Line is $\parallel$ to $y = 4x$.

56. $4y = -x + 8$
$$y = -\frac{1}{4}x + 2$$
Slope is neg. reciprocal of 4. Line is $\perp$ to $y = 4x$.

57. $\frac{1}{2}y = 2x$
$$y = 4x$$
Line coincides with $y = 4x$.

58. $-2y = \frac{1}{2}x$
$$y = -\frac{1}{4}x$$
Slope is neg. reciprocal of 4. Line is $\perp$ to $y = 4x$.

4-2 ANGLE RELATIONSHIPS IN TRIANGLES, PAGES 223–230

CHECK IT OUT!

1. **Step 1** Find $m\angle NKM$.
$m\angle KMN + m\angle MNK + m\angle NKM = 180°$
$88 + 48 + m\angle NKM = 180$
$136 + m\angle NKM = 180$
$m\angle NKM = 44°$
Step 2 Find $m\angle MJK$.
$m\angle JMK + m\angle JKM + m\angle MJK = 180°$
$44 + 104 + m\angle MJK = 180$
$148 + m\angle MJK = 180$
$m\angle MJK = 32°$

2a. Let acute ∠s be $\angle A$, $\angle B$, with $m\angle A = 63.7°$.
$m\angle A + m\angle B = 90°$
$63.7 + m\angle B = 90$
$m\angle B = 26.3°$

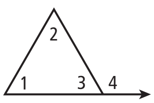
b. Let acute ∠s be $\angle C$, $\angle D$, with $m\angle C = x°$.
$m\angle C + m\angle D = 90°$
$x + m\angle D = 90$
$m\angle D = (90 - x)°$

c. Let acute ∠s be $\angle E$, $\angle F$, with $m\angle E = 48\frac{2}{5}°$.
$m\angle E + m\angle F = 90$
$48\frac{2}{5} + m\angle F = 90$
$m\angle F = 41\frac{3}{5}°$

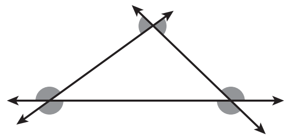
3. $m\angle ACD = m\angle ABC + m\angle BAC$
$6z - 9 = 90 + 2z + 1$
$4z = 100$
$z = 25$
$m\angle ACD = 6z - 9 = 6(25) - 9 = 141°$

4. $\angle P \cong \angle T$
$m\angle P = m\angle T$
$2x^2 = 4x^2 - 32$
$-2x^2 = -32$
$x^2 = 16$
So $m\angle P = 2x^2 = 32°$.
Since $m\angle T = m\angle P$, $m\angle T = 32°$.

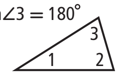
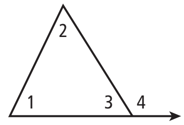
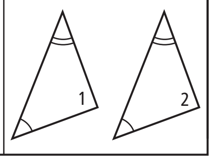
THINK AND DISCUSS

1. 

Since $\angle 3$ and $\angle 4$ are supp. ∠s, $m\angle 3 + m\angle 4 = 180°$ by def. $\angle 1 + \angle 2 + \angle 3 = 180°$ by the △ Sum Thm. By the trans. Prop. of =, $m\angle 3 + m\angle 4 = m\angle 1 + m\angle 2 + m\angle 3$. Subtract $m\angle 3$ from both sides. Then $m\angle 4 = m\angle 1 + m\angle 2$.

2. 2; 6


3.

Theorem	Words	Diagram
△ Sum Thm.	The sum of the measures of the int. ∠s of a △ is 180°.	$m\angle 1 + m\angle 2 + m\angle 3 = 180°$ 
Ext. ∠ Thm.	The measure of an ext. ∠ of a △ is = to the sum of the measures of its remote int. ∠s.	$m\angle 4 = m\angle 1 + m\angle 2$ 
Third ∠s Thm.	If 2 ∠s of 1 △ are ≅ to 2 ∠s of another △, then the third pair of ∠s are ≅.	$\angle 1 \cong \angle 2$ 

EXERCISES

GUIDED PRACTICE

1. Possible answers: think "out of the way"
2. Exterior ∠ is next to $\angle E$. So the remote interior ∠s are $\angle D$ and $\angle F$.
3. auxiliary lines
4. Think: Use △ ∠ Sum Thm.
$180 = 3y + 13 + 2y + 2 + 5y - 5$
$180 = 10y + 10$
$170 = 10y$
$y = 17$
5. Deneb: $3y + 13 = 3(17) + 13 = 64°$
Altair: $2y + 2 = 2(17) + 2 = 36°$
Vega: $5y - 5 = 5(17) - 5 = 80°$
6. $20.8 + m\angle = 90$
$m\angle = 69.2°$
7. $y + m\angle = 90$
$m\angle = (90 - y)°$
8. $24\frac{2}{3} + m\angle = 90$
$m\angle = 65\frac{1}{3}°$
9. $m\angle M + m\angle N = m\angle NPQ$
$3y + 1 + 2y + 2 = 48$
$5y + 3 = 48$
$5y = 45$
$y = 9$
$m\angle M = 3y + 1 = 3(9) + 1 = 28°$
10. $m\angle K + m\angle L = m\angle HJL$
$7x + 6x - 1 = 90$
$13x = 91$
$x = 7$
$m\angle L = 6x - 1 = 6(7) - 1 = 41°$

11. $m\angle A + m\angle B = 117$
$65 + m\angle B = 117$
$m\angle B = 52°$
$m\angle A + m\angle B + m\angle BCA = 180$
$117 + m\angle BCA = 180$
$m\angle BCA = 63°$

12. $\angle C \cong \angle F$
$m\angle C = m\angle F$
$4x^2 = 3x^2 + 25$
$x^2 = 25$
$m\angle C = 4x^2 = 100°$
$m\angle F = m\angle C = 100°$

13. $\angle S \cong \angle U$
$m\angle S = m\angle U$
$5x - 11 = 4x + 9$
$x = 20$
$m\angle S = 5x - 11$
$= 5(20) - 11$
$= 89°$
$m\angle U = m\angle S = 89°$

14. $\angle C \cong \angle Z$
$m\angle C = m\angle Z$
$4x + 7 = 3(x + 5)$
$4x + 7 = 3x + 15$
$x = 8$
$m\angle C = 4x + 7 = 4(8) + 7 = 39°$
$m\angle Z = m\angle C = 39°$

PRACTICE AND PROBLEM SOLVING

15. $m\angle A + m\angle B + m\angle P = 180$
$39 + 57 + m\angle P = 180$
$96 + m\angle P = 180$
$m\angle P = 84°$

16. $76\frac{1}{4} + m\angle = 90$
$m\angle = 13\frac{3}{4}°$

17. $2x + m\angle = 90$
$m\angle = (90 - 2x)°$

18. $56.8 + m\angle = 90$
$m\angle = 33.2°$

19. Think: Use Ext. ∠ Thm.
$m\angle W + m\angle X = m\angle XYZ$
$5x + 2 + 8x + 4 = 15x - 18$
$13x + 6 = 15x - 18$
$24 = 2x$
$x = 12$
$m\angle XYZ = 15x - 18$
$= 15(12) - 18 = 162°$

20. Think: Use Ext. ∠ Thm and subst. $m\angle C = m\angle D$.
$m\angle C + m\angle D = m\angle ABD$
$2m\angle D = m\angle ABD$
$2(6x - 5) = 11x + 1$
$12x - 10 = 11x + 1$
$x = 11$
$m\angle C = m\angle D$
$= 6x - 5$
$= 6(11) - 5 = 61°$

21. Think: Use Third ∡ Thm.
$\angle N \cong \angle P$
$m\angle N = m\angle P$
$3y^2 = 12y^2 - 144$
$-9y^2 = -144$
$y^2 = 16$
$m\angle N = 3y^2 = 3(16) = 48°$
$m\angle P = m\angle N = 48°$

22. Think: Use Third ∡ Thm.
$\angle Q \cong \angle S$
$m\angle Q = m\angle S$
$2x^2 = 3x^2 - 64$
$64 = x^2$
$m\angle Q = 2x^2 = 2(64) = 128°$
$m\angle S = m\angle Q = 128°$

23. Think: Use △ ∠ Sum Thm.
$m\angle 1 + m\angle 2 + m\angle 3 = 180$
$x + 4x + 7x = 180$
$12x = 180$
$x = 15$
$m\angle 1 = x = 15°$
$m\angle 2 = 4x = 60°$
$m\angle 3 = 7x = 105°$

24.

Statements	Reasons
1. △DEF with rt. $\angle F$	1. Given
2. $m\angle F = 90°$	2. Def. of rt. ∠
3. $m\angle D + m\angle E + m\angle F = 180°$	3. △ Sum Thm.
4. $m\angle D + m\angle E + 90° = 180°$	4. Subst.
5. $m\angle D + m\angle E = 90°$	5. Subtr. Prop.
6. $\angle D$ and $\angle E$ are comp.	6. Def. of comp. ∡

25. Proof 1:

Statements	Reasons
1. △ABC is equiangular	1. Given
2. $m\angle A = m\angle B = m\angle C$	2. Def. of equilangular
3. $m\angle A + m\angle B + m\angle C = 180°$	3. △ Sum Thm.
4. $m\angle A + m\angle A + m\angle A = 180°$ $m\angle B + m\angle B + m\angle B = 180°$ $m\angle C + m\angle C + m\angle C = 180°$	4. Subst. prop
5. $3m\angle A = 180°$, $3m\angle B = 180°$, $3m\angle C = 180°$	5. Simplify.
6. $m\angle A = 60°$, $m\angle B = 60°$, $m\angle C = 60°$	6. Div. Prop. of =

Proof 2:
$\angle A$, $\angle B$, and $\angle C$ are all congruent, so their measures are equal. The sum of the three ∠ measures is 180°, by △ Sum Thm. Therefore, 3 • (common ∠ measure) = 180°. So the common ∠ measure is 60°. That is, $m\angle A = m\angle B = m\angle C = 60°$.

26. **Step 1** Write an equation.

$m\angle 1 = 1\frac{1}{4}m\angle 2$

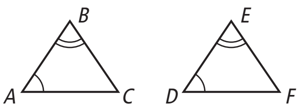
Step 2 Since the acute $\angle$s of a rt. $\triangle$ are comp. write and solve another equation.

$$m\angle 1 + m\angle 2 = 90$$
$$1\frac{1}{4}m\angle 2 + m\angle 2 = 90$$
$$\frac{9}{4}m\angle 2 = 90$$
$$m\angle 2 = \frac{4}{9}(90) = 40°$$

Step 3 Find the larger acute $\angle$, $m\angle 1$.

$$m\angle 1 = 1\frac{1}{4}m\angle 2 = \frac{5}{4}(40) = 50°$$

27.



Statements	Reasons
1. $\triangle ABC$, $\triangle DEF$, $\angle A \cong \angle D$, $\angle B \cong \angle E$	1. Given
2. $m\angle A + m\angle B + m\angle C = 180°$	2. $\triangle$ Sum Thm.
3. $m\angle C = 180° - m\angle A - m\angle B$	3. Subtr. Prop. of =
4. $m\angle D + m\angle E + m\angle F = 180°$	4. $\triangle$ Sum Thm.
5. $m\angle F = 180° - m\angle D - m\angle E$	5. Subtr. Prop. of =
6. $m\angle A = m\angle D$, $m\angle B = m\angle E$	6. Def. of $\cong$ $\angle$s
7. $m\angle F = 180° - m\angle A - m\angle B$	7. Subst.
8. $m\angle F = m\angle C$	8. Trans. Prop. of =
9. $\angle F \cong \angle C$	9. Def. of $\cong$ $\angle$s

28.

Statements	Reasons
1. $\triangle ABC$ with ext. $\angle ACD$	1. Given
2. $m\angle A + m\angle B + m\angle ACB = 180°$	2. $\triangle$ Sum Thm.
3. $m\angle ACB + m\angle ACD = 180°$	3. Lin. Pair Thm.
4. $m\angle ACD = 180° - m\angle ACB$	4. Subtr. Prop. of =
5. $m\angle ACD = (m\angle A + m\angle B + m\angle ACB) - m\angle ACB$	5. Subst.
6. $m\angle ACD = m\angle A + m\angle B$	7. Simplify.

29. Think: Use Alt. Int. $\angle$s Thm.

$$m\angle WUX + m\angle UXZ = 180$$
$$m\angle WUX + 90 = 180$$
$$m\angle WUX = 90°$$

So $\triangle UWX$ is a rt. $\triangle$.

$$m\angle UXW + m\angle XWU = 90$$
$$m\angle UXW + 54 = 90$$
$$m\angle UXW = 36°$$

30. $\angle XWU$, $\angle UWY$, and $\angle YWV$ are supp. $\angle$s.

$$m\angle XWU + m\angle UWY + m\angle YWV = 180$$
$$54 + m\angle UWY + 78 = 180$$
$$m\angle UWY + 132 = 180$$
$$m\angle UWY = 48°$$

31. Think: Use Third $\angle$s Thm.

$\angle WUY \cong \angle ZXY$
$\angle UYW \cong \angle XYZ$
$\angle WZX \cong \angle UWY$
$m\angle WZX = m\angle UWY = 48°$

32. $\angle XYZ$ and $\angle WZX$ are acute $\angle$s in a rt. $\triangle$.

$$m\angle XYZ + m\angle WZX = 90$$
$$m\angle XYZ + 48 = 90$$
$$m\angle XYZ = 42°$$

33. Let $\angle 1$, $\angle 2$, and $\angle 3$ be internal $\angle$s. Let $\angle 4$, $\angle 5$, and $\angle 6$ be external $\angle$s.
Think: Use Ext. $\angle$ Thm.
$m\angle 4 = m\angle 1 + m\angle 2$
$m\angle 1 = m\angle 2 = 60°$
So $m\angle 4 = 60 + 60 = 120°$.
Likewise, $m\angle 5 = m\angle 6 = 120°$.
Ext. $\angle$ sum $= m\angle 4 + m\angle 5 + m\angle 6 = 360°$

34. Think: Use Third $\angle$s Thm.
$\angle SRQ \cong \angle RST$
$m\angle SRQ = m\angle RST = 37.5°$

35. Let acute $\angle$ measures be $x°$ and $4x°$.

$$x + 4x = 90$$
$$5x = 90$$
$$x = 18$$

Smallest $\angle$ measure is $x° = 18°$.

36a. hypotenuse

b. $x° + y° + 90° = 180°$

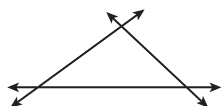
c. $x° + y° = 90$
x and y are comp. $\angle$ measures.

d. $z° = x° + 90°$

e.
$x + y = 90$
$37 + y = 90$
$y = 53°$

$z = x + 90$
$z = 37 + 90$
$z = 127°$

37.



The ext. $\angle$s at the same vertex of a $\triangle$ are vert. $\angle$s. Since vert. $\angle$s are $\cong$, the 2 ext. $\angle$s have the same measure.

38.

Statements	Reasons
1. $\overline{AB} \perp \overline{BD}$, $\overline{BD} \perp \overline{CD}$, $\angle A \cong \angle C$	1. Given
2. $\angle ABD$ and $\angle CDB$ are rt. $\angle$s	2. Def. of $\perp$ lines
3. $m\angle ABD = m\angle CBD$	3. Def. of rt. $\angle$s
4. $\angle ABD \cong \angle CDB$	4. Rt. $\angle$ $\cong$ Thm.
5. $\angle ADB \cong \angle CBD$	5. Third $\angle$s Thm.
6. $\overline{AD} \parallel \overline{CB}$	6. Conv. of Alt. Int. $\angle$s Thm.

39. Check students' sketches. Ext. $\angle$ measures = sums of remote int. $\angle$ measures: 155°, 65°, and 140°.

40a. $m\angle FCE = \frac{1}{2}m\angle DCE$
$= \frac{1}{2}(90) = 45°$
$m\angle FCB = \frac{1}{2}m\angle FCE$
$= \frac{1}{2}(45) = 22.5°$

b. $m\angle CBE + m\angle BEC + m\angle BCE = 180$
$m\angle CBE + 90 + 22.5 = 180$
$m\angle CBE + 112.5 = 180$
$m\angle CBE = 67.5°$

TEST PREP

41. C
$128 = 71 + x$
$x = 57$

42. F
$(2s + 10) + 58 + 66 = 180$
$2s + 134 = 180$
$2s = 46$
$s = 23$

43. D
$m\angle A + m\angle B = m\angle BCD$
$m\angle B = m\angle BCD - m\angle A$

44. Let $2x$, $3x$, and $4x$ represent the ∠ measures. The sum of the ∠ measures of a △ is 180°, so $2x + 3x + 4x = 180°$. Solving the eqn. for the value of x, yields $x = 20$. Find each measure by subsituting 20 for x in each expression. $2x = 2(20) = 40$; $3x = 3(20) = 60$; $4x = 4(20) = 80$. Since all of the ∡ measure less than 90°, they are all acute ∡ by def. Thus the △ is acute.

CHALLENGE AND EXTEND

45. $117 = (2y^2 + 7) + (61 - y^2)$
$117 = y^2 + 68$
$49 = y^2$
$y = 7$ or -7

46. A rt. △ is formed. The 2 same-side int. ∡ are supp., so the 2 ∡ formed by their bisectors must be comp. That means the remaining ∠ of the △ must measure 90°.

47. Since an ext. ∠ is = to a sum of 2 remote int. ∡, it must be greater than either ∠. Therefore, it cannot be ≅ to a remote int. ∠.

48. Possible sets of ∠ measures:
(30, 30, 120), (30, 60, 90), (60, 60, 60)
Probability $= \frac{2}{3}$

49. Let $m\angle A = x°$.
$m\angle B = 1\frac{1}{2}(x) - 5$
$m\angle C = 2\frac{1}{2}(x) - 5$
$m\angle A + m\angle B + m\angle C = 180$
$x + 1\frac{1}{2}(x) - 5 + 2\frac{1}{2}(x) - 5 = 180$
$5x - 10 = 180$
$5x = 190$
$x = 38$
$m\angle A = x° = 38°$

SPIRAL REVIEW

50.

x	−2	0	1	4
$f(x)$	−10	−4	−1	8

51.

x	−2	0	1	4
$f(x)$	5	1	2	17

52.

x	−2	0	1	4
$f(x)$	30	14	9	6

53. Use Seg. Add. Post.
$MN + NP = MP$
$4 + NP = 6$
$NP = 2$ in.
$NP + PQ = NQ$
$2 + 4 = NQ$
$NQ = 6$ in.

54. $\overline{AD} \cong \overline{CD} \ncong \overline{AC}$
Isosc.

55. $\overline{BD}$, $\overline{CD}$, $\overline{BC}$ are ≇
Scalene

56. $\overline{AB}$, $\overline{AD}$, $\overline{BD}$ are ≇
Scalene

57. $\overline{AD} \cong \overline{CD} \cong \overline{AC}$
Equilateral

4-3 CONGRUENT TRIANGLES, PAGES 231–237

CHECK IT OUT!

1. Angles: $\angle L \cong \angle E$, $\angle M \cong \angle F$, $\angle N \cong \angle G$, $\angle P \cong \angle H$
Sides: $\overline{LM} \cong \overline{EF}$, $\overline{MN} \cong \overline{FG}$, $\overline{NP} \cong \overline{GH}$, $\overline{LP} \cong \overline{EH}$

2a. $\overline{AB} \cong \overline{DE}$
$2x - 2 = 6$
$2x = 8$
$x = 4$

b. Since the acute ∡ of a rt. △ are comp.
$m\angle B + m\angle C = 90$
$53 + m\angle C = 90$
$m\angle C = 37°$
$\angle F \cong \angle C$
$m\angle F = m\angle C = 37°$

3.

Statements	Reasons
1. $\angle A \cong \angle D$	1. Given
2. $\angle BCA \cong \angle ECD$	2. Vert. $\angle$s are $\cong$.
3. $\angle ABC \cong \angle DEC$	3. Third $\angle$s Thm.
4. $\overline{AB} \cong \overline{DE}$	4. Given
5. $\overline{AD}$ bisects $\overline{BE}$, and $\overline{BE}$ bisects $\overline{AD}$.	5. Given
6. $\overline{BC} \cong \overline{EC}$, $\overline{AC} \cong \overline{BC}$	6. Def. of bisector
7. $\triangle ABC \cong \triangle DEC$	7. Def. of $\cong$ $\triangle$s

4.

Statements	Reasons
1. $\overline{JK} \parallel \overline{ML}$	1. Given
2. $\angle KJN \cong \angle MLN$, $\angle JKN \cong \angle LMN$	2. Alt. Int. $\angle$s Thm.
3. $\angle JNK \cong \angle LNM$	3. Vert. $\angle$s Thm.
4. $\overline{JK} \cong \overline{ML}$	4. Given
5. $\overline{MK}$ bisects $\overline{JL}$, and $\overline{JL}$ bisects $\overline{MK}$.	5. Given
6. $\overline{JN} \cong \overline{LN}$, $\overline{MN} \cong \overline{KN}$	6. Def. of bisector
7. $\triangle JKN \cong \triangle MLN$	7. Def. of $\cong$ $\triangle$s

THINK AND DISCUSS

1. Measure all the sides and all the $\angle$s. The trusses are the same size if all the corr. sides and $\angle$s are $\cong$.

2.

$\triangle PQR \cong \triangle LMN$

Angles: $\angle P \cong \angle L$, $\angle Q \cong \angle M$, $\angle R \cong \angle N$

Sides: $\overline{PQ} \cong \overline{LM}$, $\overline{QR} \cong \overline{MN}$, $\overline{PR} \cong \overline{LN}$

EXERCISES

GUIDED PRATICE

1. You find the $\angle$s and sides that are in the same, or matching, places in the 2 $\triangle$s.

2. $\angle B$

3. $\overline{LM}$

4. $\overline{RT}$

5. $\angle M$

6. $\overline{NM}$

7. $\angle R$

8. $\angle T$

9.
$$\overline{JK} \cong \overline{FG}$$
$$JK = FG$$
$$3y - 15 = 12$$
$$3y = 27$$
$$y = 9$$
$$KL = y = 9$$

10.
$$\angle G \cong \angle K$$
$$m\angle G = m\angle K$$
$$4x - 20 = 108$$
$$4x = 128$$
$$x = 32$$

11.

Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	1. Given
2. $\angle ABE \cong \angle CDE$, $\angle BAE \cong \angle DCE$	2. Alt. Int. $\angle$ Thm.
3. $\overline{AB} \cong \overline{CD}$	3. Given
4. E is the mdpt. of $\overline{AC}$ and $\overline{BD}$	4. Given
5. $\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$	5. Def. of mdpt.
6. $\angle AEB \cong \angle CED$	6. Vert. $\angle$ Thm
7. $\triangle ABE \cong \triangle CDE$	7. Def. of $\cong$ $\triangle$s

PRACTICE AND PROBLEM SOLVING, PAGES 235–236

12.

Statements	Reasons
1. $\angle UST \cong \angle RST$, $\angle U \cong \angle R$	1. Given
2. $\angle STU \cong \angle STR$	2. Third $\angle$s Thm.
3. $\overline{SU} \cong \overline{SR}$	3. Given
4. $\overline{ST} \cong \overline{ST}$	4. Reflex. Prop. of $\cong$
5. $\overline{TU} \cong \overline{TR}$	5. Given
6. $\triangle RTS \cong \triangle UTS$	6. Def. of $\cong$ $\triangle$s

13. $\overline{LM}$

14. $\overline{CF}$

15. $\angle N$

16. $\angle D$

17.
$$\angle ADB \cong \angle CDB$$
$$m\angle ADB = m\angle CDB$$
$$4x + 10 = 90$$
$$4x = 80$$
$$x = 20$$
$$m\angle C = x + 11 = 31°$$

18.
$$\overline{AB} \cong \overline{CB}$$
$$AB = CB$$
$$y - 7 = 12$$
$$y = 19$$

19.

Statements	Reasons
1. $\angle N \cong \angle R$	1. Given
2. $\overline{MP}$ bisects $\angle NMR$	2. Given
3. $\angle NMP \cong \angle RMP$	3. Def. of $\angle$ bisector
4. $\angle NPM \cong \angle RPM$	4. Third $\angle$s Thm.
5. P is the mdpt. of $\overline{NR}$	5. Given
6. $\overline{PN} \cong \overline{PR}$	6. Def. of mdpt.
7. $\overline{MN} \cong \overline{MR}$	7. Given
8. $\overline{MP} \cong \overline{MP}$	8. Reflex. Prop. of $\cong$
9. $\triangle MNP \cong \triangle MRP$	9. Def. of $\cong$ $\triangle$s

20.

Statements	Reasons
1. $\angle ADC$ and $\angle BCD$ are rt. $\angle$s	1. Given
2. $\angle ADC \cong \angle BCD$	2. Rt. $\angle$ $\cong$ Thm.
3. $\angle DAC \cong \angle CBD$	3. Given
4. $\angle ACD \cong \angle BDC$	4. Third $\angle$s Thm.
5. $\overline{AC} \cong \overline{BD}$, $\overline{AD} \cong \overline{BC}$	5. Given
6. $\overline{DC} \cong \overline{DC}$	6. Reflex. Prop. of $\cong$
7. $\triangle ADC \cong \triangle BCD$	7. Def. of $\cong$ $\triangle$s

21. $\triangle GSR \cong \triangle KPH$,
$\triangle SRG \cong \triangle PHK$
$\triangle RSG \cong \triangle HPK$,

22. $RVUTS \cong VWXZY$

23.
$$\overline{AB} \cong \overline{DE}$$
$$AB = DE$$
$$2x - 10 = x + 20$$
$$x = 30$$
$$AB = 2x - 10$$
$$= 2(30) - 10 = 50$$

24.
$$\angle L \cong \angle P$$
$$m\angle L = m\angle P$$
$$x^2 + 10 = 2x^2 + 1$$
$$9 = x^2$$
$$m\angle L = x^2 + 10$$
$$= 9 + 10 = 19°$$

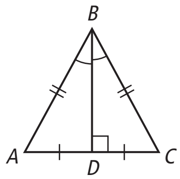
25.
$$\overline{BC} \cong \overline{QR}$$
$$BC = QR$$
$$6x + 5 = 5x + 7$$
$$x = 2$$
$$BC = 6x + 5$$
$$= 6(2) + 5 = 17$$

26a. $\overline{KL} \cong \overline{ML}$ by the def. of a square.

b.

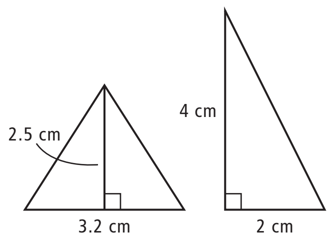
Statements	Reasons
1. $JKLM$ is a square.	1. Given
2. $\overline{KL} \cong \overline{ML}$	2. Def. of a square
3. $\overline{JL}$ and $\overline{MK}$ are $\perp$ bisectors of each other.	3. Given
4. $\overline{MN} \cong \overline{KN}$	4. Def. of bisector
5. $\overline{NL} \cong \overline{NL}$	5. Reflex. Prop. of $\cong$
6. $\angle MNL$ and $\angle KNL$ are rt. $\angle$s.	6. Def. of $\perp$
7. $\angle MNL \cong \angle KNL$	7. Rt. $\angle \cong$ Thm.
8. $\angle NML \cong \angle NKL$	8. Given
9. $\angle NLM \cong \angle NLK$	9. Third $\angle$s Thm.
10. $\triangle NML \cong \triangle NKL$	10. Def. of $\cong \triangle$s

27.



Statements	Reasons
1. $\overline{BD} \perp \overline{AC}$	1. Given
2. $\angle ADB$ and $\angle CDB$ are rt. $\angle$s.	2. Def. of $\perp$
3. $\angle ADB \cong \angle CDB$	3. Rt. $\angle \cong$ Thm.
4. $\overline{BD}$ bisects $\angle ABC$.	4. Given
5. $\angle ABD \cong \angle CBD$	5. Def. of bisector
6. $\angle A \cong \angle C$	6. Third $\angle$s Thm.
7. $\overline{AB} \cong \overline{CB}$	7. Given
8. $\overline{BD} \cong \overline{BD}$	8. Reflex. Prop. of $\cong$
9. D is the mdpt. of $\overline{AC}$.	9. Given
10. $\overline{AD} \cong \overline{CD}$	10. Def. of mdpt.
11. $\triangle ABD \cong \triangle CBD$	12. Def. of $\cong \triangle$s

28. Possible answer:



29. Solution A is incorrect. $\angle E \cong \angle M$, so $m\angle E = 46°$.

30. Yes; by the Third $\angle$s Thm., $\angle K \cong \angle W$, so all 6 pairs of corr. parts are $\cong$. Therefore, the $\triangle$s are $\cong$.

TEST PREP

31. B
Matching up $\angle$s, $\triangle ABC \cong \triangle FDE$.

32. G

$$\angle N \cong \angle S \qquad \angle M \cong \angle R$$
$$m\angle N = m\angle S \qquad m\angle M = m\angle R$$
$$62 = 2x + 8 \qquad 58 = 3y - 2$$
$$54 = 2x \qquad 60 = 3y$$
$$x = 27 \qquad y = 20$$

33. D
$$m\angle Y = 180 - (m\angle X + m\angle Z)$$
$$= 180 - (m\angle A + m\angle C)$$
$$= 180 - 60.9 = 119.1°$$

34. J
$$P = MN + NR + RM$$
$$= SP + QP + SR + RQ$$
$$= 33 + 30 + 10 + 24 = 97$$

CHALLENGE AND EXTEND

35. $P = TU + UV + VW + TW$
$149 = 6x + 7x + 3 + 9x - 8 + 8x - 11$
$149 = 30x - 16$
$165 = 30x$
$x = 5.5$
Yes; $UV = WV = 41.5$, and $UT = WT = 33$.
$TV = TV$ by the Reflex. Prop. of =. It is given that $\angle VWT \cong \angle VUT$ and $\angle WTV \cong \angle UTV$.
$\angle WVT = \angle UVT$ by the Third ≜ Thm. Thus $\triangle TUV \cong \triangle TWV$ by the def. of ≅ ≜.

36. $\angle E \cong \angle A$
$m\angle E = m\angle A$
$y^2 - 10 = 90$
$y^2 = 100$
$m\angle D = m\angle H$
$= 2y^2 - 132$
$= 2(100) - 132 = 68°$

37.

Statements	Reasons
1. $\overline{RS} \cong \overline{RT}$; $\angle S \cong \angle T$	1. Given
2. $\overline{ST} \cong \overline{TS}$	2. Reflex. Prop. of ≅
3. $\angle T \cong \angle S$	3. Sym. Prop. of ≅
4. $\angle R \cong \angle R$	4. Reflex. Prop. of ≅
5. $\triangle RST \cong \triangle RTS$	5. Def. of ≅ ≜

SPIRAL REVIEW

38. P(both even) = P(cube 1 even) • P(cube 1 even)
$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

39. P(sum is 5) = P(1, 4) + P(2, 3) + P(3, 2) + P(4, 1)
$= \frac{4}{36} = \frac{1}{9}$

40. acute

41. rt.

42. obtuse

43. Step 1 Find x.
$3x + 20 + 4x + x + 16 = 180$
$8x + 36 = 180$
$x = 18$
Step 2 Find $m\angle Q$.
$m\angle Q = 4x = 72°$

44. $m\angle P = 3x + 20 = 74°$

45. $m\angle QRS = m\angle P + m\angle Q$
$= 72 + 74 = 146°$

READY TO GO ON? PAGE 239

1. rt. △, since $\angle ACB$ is rt. ∠

2. equiangular, since $m\angle BAD = 30 + 30 = 60°$
$= m\angle B = m\angle ADB$

3. obtuse, since $m\angle ADE = m\angle B + m\angle BAD = 120°$

4. isosc., since $PQ = QR = 5$, $PR = 8.7$

5. equilateral, since $PR = RS = PS = 5$

6. scalene, since $PQ = 8.7$, $QS = 5 + 5 = 10$, $PS = 5$

7. $m\angle M + m\angle N = m\angle NLK$
$6y + 3 + 84 = 151 - 2y$
$8y = 64$
$y = 8$
$m\angle M = 6y + 3 = 51°$

8. $m\angle C + m\angle D = m\angle ABC$
$90 + 5x = 20x - 15$
$105 = 15x$
$x = 7$
$m\angle ABC = 20x - 15 = 125°$

9. $m\angle RTP = m\angle R + m\angle T = 55 + 37 = 92°$

10. $\overline{EF}$

11. $\overline{JL}$

12. $\angle E$

13. $\angle L$

14. $\overline{PR} \cong \overline{SU}$
$PR = SU$
$14 = 3m + 2$
$12 = 3m$
$m = 4$
$PQ = 2m + 1 = 9$

15. $\angle S \cong \angle P$
$m\angle S = m\angle P$
$2y = 46$
$y = 23$

16.

Statements	Reasons
1. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	1. Given
2. $\angle BAD \cong \angle CDA$	2. Alt. Int. ≜ Thm.
3. $\overline{AC} \perp \overline{CD}$, $\overline{DB} \perp \overline{AB}$	3. Given.
4. $\angle ACD$ and $\angle DBA$ are rt. ≜	4. Def. of ⊥
5. $\angle ACD \cong \angle DBA$	5. Rt. ∠ ≅ Thm.
6. $\angle CAD \cong \angle BDA$	6. Third ≜ Thm.
7. $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{DB}$	7. Given
8. $\overline{AD} \cong \overline{DA}$	8. Reflex. Prop. of ≅
9. $\triangle ACD \cong \triangle DBA$	9. Def. of ≅ ≜

4-4 TRIANGLE CONGRUENCE: SSS AND SAS, PAGES 242–249

CHECK IT OUT!

1. It is given that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$. By the Reflex. Prop. of ≅, $\overline{AC} \cong \overline{CA}$. So $\triangle ABC \cong \triangle CDA$ by SSS.

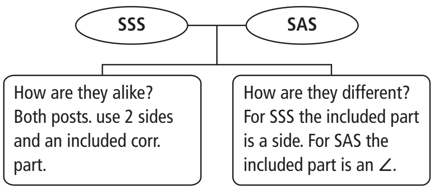
2. It is given that $\overline{AB} \cong \overline{BD}$ and $\angle ABC \cong \angle DBC$. By Reflex. Prop. of ≅, $\overline{BC} \cong \overline{BC}$. So $\triangle ABC \cong \triangle DBC$ by SAS.

3. $DA = 3t + 1$
$= 3(4) + 1 = 13$
$DC = 4t - 3$
$= 4(4) - 3 = 13$
$m\angle ADB = 32°$
$m\angle CDB = 2t^2$
$= 2(4)^2 = 32°$
$\overline{DA} \cong \overline{DC}$, $\overline{DB} \cong \overline{DB}$, and $\angle ADB \cong \angle CDB$
So $\triangle ADB \cong \triangle CDB$ by SAS.

4.

Statements	Reasons
1. $\overline{QR} \cong \overline{QS}$	1. Given
2. $\overrightarrow{QP}$ bisects $\angle RQS$	2. Given
3. $\angle RQP \cong \angle SQP$	3. Def. of $\angle$ bisector
4. $\overline{QP} \cong \overline{QP}$	4. Reflex. Prop. of $\cong$
5. $\triangle RQP \cong \triangle SQP$	5. SAS Steps 1, 3, 4

THINK AND DISCUSS

1. Show that all six pairs of corr. parts are $\cong$; SSS; SAS
2. The SSS and SAS Post. are methods for proving $\triangle$s $\cong$ without having to prove $\cong$ of all 6 corr. parts.
3. 

EXERCISES

GUIDED PRACTICE

1. $\angle T$
2. It is given that $\overline{DA} \cong \overline{BC}$ and $\overline{AB} \cong \overline{CD}$. $\overline{BD} \cong \overline{DB}$ by the Reflex. Prop. of $\cong$. Thus $\triangle ABD \cong \triangle CDB$ by SSS.
3. It is given that $\overline{MN} \cong \overline{MQ}$ and $\overline{NP} \cong \overline{QP}$. $\overline{MP} \cong \overline{MP}$ by the Relex. Prop. of $\cong$. Thus $\triangle MNP \cong \triangle MQP$ by SSS.
4. It is given that $\overline{JG} \cong \overline{LG}$, and $\overline{GK} \cong \overline{GH}$. $\angle JGK \cong \angle LGH$ by the Vert. $\angle$s Thm. So $\triangle JGK \cong \triangle LGH$ by SAS.
5. When $x = 4$, $HI = GH = 3$, and $IJ = GJ = 5$. $\overline{HJ} \cong \overline{HJ}$ by the Reflex. Prop. of $\cong$. Therefore, $\triangle GHJ \cong \triangle IHJ$ by SSS.
6. When $x = 18$, $RS = UT = 61$, and $m\angle SRT = m\angle UTR = 36°$. $\overline{RT} \cong \overline{TR}$ by the Reflex. Prop. of $\cong$. So $\triangle RST \cong \triangle TUR$ by SAS.

7.

Statements	Reasons
1. $\overline{JK} \cong \overline{ML}$	1. Given
2. $\angle JKL \cong \angle MLK$	2. Given
3. $\overline{KL} \cong \overline{LK}$	3. Reflex. Prop. of $\cong$
4. $\triangle JKL \cong \triangle MLK$	4. SAS Steps 1, 2, 3

PRACTICE AND PROBLEM SOLVING

8. It is given that $BC = ED = 4$ in. and $BD = EC = 3$ in. So by the def. of $\cong$, $\overline{BC} \cong \overline{ED}$, and $\overline{BD} \cong \overline{EC}$. $\overline{DC} \cong \overline{CD}$ by the Reflex. Prop. of $\cong$. Thus $\triangle BCD \cong \triangle EDC$ by SSS.
9. It is given that $\overline{KJ} \cong \overline{LJ}$ and $\overline{GK} \cong \overline{GL}$. $\overline{GJ} \cong \overline{GJ}$ by the Reflex. Prop. of $\cong$. So $\triangle GJK \cong \triangle GJL$ by SSS.
10. It is given that $\angle C$ and $\angle B$ are rt. $\angle$s and $\overline{EC} \cong \overline{DB}$. $\angle C \cong \angle B$ by the Rt. $\angle \cong$ Thm. $\overline{CB} \cong \overline{BC}$ by the Reflex. Prop. of $\cong$. So $\triangle ECB \cong \triangle DBC$ by SAS.
11. When $y = 3$, $NQ = NM = 3$, and $QP = MP = 4$. So by the def. of $\cong$, $\overline{NQ} \cong \overline{NM}$, and $\overline{QP} \cong \overline{MP}$. $m\angle M = m\angle Q = 90°$, so $\angle M \cong \angle Q$ by the def. of $\cong$. Thus $\triangle MNP \cong \triangle QNP$ by SAS.
12. When $t = 5$, $YZ = 24$, $ST = 20$, and $SU = 22$. So by the def. of $\cong$, $\overline{XY} \cong \overline{ST}$, $\overline{YZ} \cong \overline{TU}$, and $\overline{XZ} \cong \overline{SU}$. This $\triangle XYZ \cong \triangle STU$ by SSS.

13.

Statements	Reasons
1. B is mdpt. of $\overline{DC}$	1. Given
2. $\overline{DB} \cong \overline{CB}$	2. Def. of mdpt.
3. $\overline{AB} \perp \overline{DC}$	3. Given
4. $\angle ABD$ and $\angle ABC$ are rt. $\angle$s	4. Def. of $\perp$
5. $\angle ABD \cong \angle ABC$	5. Rt. $\angle \cong$ Thm.
6. $\overline{AB} \cong \overline{AB}$	6. Reflex. Prop. of $\cong$
7. $\triangle ABD \cong \triangle ABC$	7. SAS Steps 2, 5, 6

14. SAS (with Reflex. Prop of $\cong$)
15. SAS (with Vert. $\angle$s Thm.)
16. neither
17. neither

18a. To use SSS, you need to know that $\overline{AB} \cong \overline{DE}$ and $\overline{CB} \cong \overline{CE}$.

b. To use SAS, you need to know that $\overline{CB} \cong \overline{CE}$.

19. $QS = \sqrt{1^2 + 2^2} = \sqrt{5}$
$SR = \sqrt{4^2 + 0^2} = 4$
$QR = \sqrt{3^2 + 2^2} = \sqrt{13}$
$TV = \sqrt{1^2 + 2^2} = \sqrt{5}$
$VU = \sqrt{4^2 + 0^2} = 4$
$TU = \sqrt{3^2 + 2^2} = \sqrt{13}$
The $\triangle$s are $\cong$ by SSS.

20. $AB = \sqrt{1^2 + 4^2} = \sqrt{17}$
$BC = \sqrt{4^2 + 3^2} = 5$
$AC = \sqrt{5^2 + 1^2} = \sqrt{26}$
$DE = \sqrt{1^2 + 4^2} = \sqrt{17}$
$EF = \sqrt{4^2 + 3^2} = 5$
$DF = \sqrt{4^2 + 0^2} = 4$
The $\triangle$s are not $\cong$.

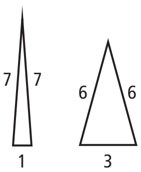
21.

Statements	Reasons
1. $\angle ZVY \cong \angle WYV$, $\angle ZVW \cong \angle WYZ$	1. Given
2. $m\angle ZVY = m\angle WYV$, $m\angle ZVW = m\angle WYZ$	2. Def. of $\cong$
3. $m\angle ZVY + m\angle ZVW = m\angle WYV + m\angle WYZ$	3. Add. Prop. of =
4. $m\angle WVY = m\angle ZYV$	4. $\angle$ Add. Post.
5. $\angle WVY \cong \angle ZYV$	5. Def. of $\cong$
6. $\overline{WV} \cong \overline{YZ}$	6. Given
7. $\overline{VY} \cong \overline{YV}$	7. Reflex. Prop. of $\cong$
8. $\triangle ZVY \cong \triangle WYV$	8. SAS Steps 6, 5, 7

22a. Measure $\overline{AB}$ and $\overline{AC}$ on 1 truss and measure $\overline{DE}$ and $\overline{DF}$ on the other. If $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$, then the trusses are $\cong$ by SAS.

b. 3.5 ft; by the Pyth. Thm., BC ≈ 3.5 ft. Since the △s are congruent, $\overline{EF} \cong \overline{BC}$.

23.



24. $AB = AC$ $\qquad$ $BC = DC$

$4x = 6x - 11$ $\qquad$ $x + 4 = 5x - 7$

$11 = 2x$ $\qquad$ $11 = 2x$

$x = 5.5$ $\qquad$ $x = 5.5$ ✓

By the def. of $\cong$, $AB \cong BD$, and $BC \cong DC$. $AC \cong AC$ by the Reflex. Prop. of $\cong$. Thus $\triangle ABC \cong \triangle ADC$ by SSS.

25. Measure the lengths of the logs. If the lengths of the logs in 1 wing deflector match the lengths of the logs in the other wing deflector, the △s will be $\cong$ by SAS or SSS.

26. Yes; if the △s have the same 2 side lengths and the same included $\angle$ measure, the △s are $\cong$ by SAS.

27. Check students' constructions; yes; if each side is $\cong$ to the corr. side of the second △, they can be in any order.

TEST PREP

28. C
In I and III, two sides are congruent with an congruent angle in between so I and III are similar by SAS.

29. G
SAS proves $\triangle ABC \cong \triangle ADC$, so

$$\begin{aligned} AB + BC + CD + DA &= AB + CD + CD + AB \\ &= 12.1 + 7.8 + 7.8 + 12.1 \\ &= 39.8 \text{ cm} \end{aligned}$$

30. A
$\angle F$ and $\angle J$ are the included ∠s, so $\angle F \cong \angle J$ proves SAS.

31. J

$$\begin{aligned} \overline{EF} &\cong \overline{EH} \\ EF &= EH \\ 4x + 7 &= 6x - 4 \\ 11 &= 2x \\ x &= 5.5 \end{aligned}$$

CHALLENGE AND EXTEND

32.

Statements	Reasons
1. Draw $\overline{DB}$.	1. Through any 2 pts. there is exactly one line.
2. $\angle ADC$ and $\angle BCD$ are supp.	2. Given
3. $\overline{AD} \parallel \overline{CB}$	3. Conv. of Same-Side Int. ∠s Thm.
4. $\angle ADB \cong \angle CBD$	4. Alt. Int. ∠s Thm.
5. $\overline{AD} \cong \overline{CB}$	5. Given
6. $\overline{DB} \cong \overline{BD}$	6. Reflex Prop. of $\cong$
7. $\triangle ADB \cong \triangle CBD$	7. SAS *Steps 5, 4, 6*

33.

Statements	Reasons
1. $\angle QPS \cong \angle TPR$	1. Given
2. $\angle RPS \cong \angle RPS$	2. Reflex. Prop. of $\cong$
3. $\angle QPR \cong \angle TPS$	3. Subst. Prop. of $\cong$
4. $\overline{PQ} \cong \overline{PT}$, $\overline{PR} \cong \overline{PS}$	4. Given
5. $\triangle PQR \cong \triangle PTS$	5. SAS *Steps 3, 4*

34.

$$\begin{aligned} m\angle FKJ + m\angle KFJ + m\angle FJK &= 180 \\ 2x + 3x + 10 + 90 &= 180 \\ 5x &= 80 \\ x &= 16 \end{aligned}$$

$KJ = HJ = 72$, so $\overline{KJ} \cong \overline{HJ}$ by the def. of $\cong$. $\angle FJK \cong \angle FJH$ by the Rt. $\angle \cong$ Thm. $\overline{FJ} \cong \overline{FJ}$ by the Reflex. Prop. of $\cong$. So $\triangle FJK \cong \triangle FJH$ by SAS.

35.

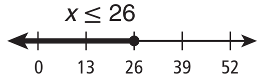
$$\begin{aligned} m\angle KFJ &= m\angle HFJ \\ 2x + 6 &= 3x - 21 \\ 27 &= x \end{aligned}$$

$FK = FH = 171$, so $\overline{FK} \cong \overline{FH}$ by the def. of $\cong$. $\angle KFJ \cong \angle HFJ$ by the def. of $\angle$ bisector. $\overline{FJ} \cong \overline{FJ}$ by the Reflex. Prop. of $\cong$. So $\triangle FJK \cong \triangle FJH$ by SAS.

SPIRAL REVIEW

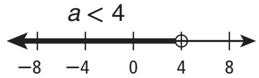
36.

$$\begin{aligned} \frac{x}{2} - 8 &\le 5 \\ x - 16 &\le 10 \\ x &\le 26 \end{aligned}$$



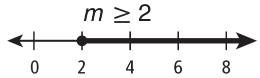
37.

$$\begin{aligned} 2a + 4 &> 3a \\ 4 &> a \\ a &< 4 \end{aligned}$$



38.

$$\begin{aligned} -6m - 1 &\le -13 \\ 12 &\le 6m \\ m &\ge 2 \end{aligned}$$



39. $4x - 7 = 21$ Given
$4x - 7 + 7 = 21 + 7$ Add. Prop. of =
$4x = 28$ Simplify.
$\frac{4x}{4} = \frac{28}{4}$ Div. Prop. of =
$x = 7$ Simplify.

40. $\frac{a}{4} + 5 = -8$ Given
$\frac{a}{4} + 5 - 5 = -8 - 5$ Subtr. Prop. of =
$\frac{a}{4} = -13$ Simplify.
$4\left(\frac{a}{4}\right) = 4(-13)$ Multi. Prop. of =
$a = -52$ Simplify.

41. $6r = 4r + 10$ Given
$6r - 4r = 4r - 4r + 10$ Subtr. Prop. of =
$2r = 10$ Simplify.
$\frac{2r}{2} = \frac{10}{2}$ Div. Prop. of =
$r = 5$ Simplify.

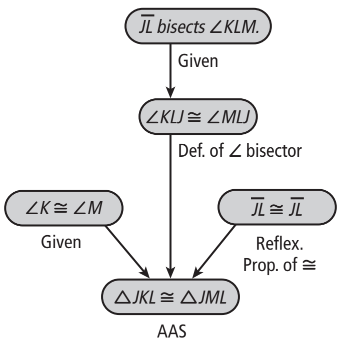
42. $\angle H \cong \angle F$
$m\angle H = m\angle F$
$x + 24 = 110$
$x = 86$

43. $m\angle FGE = m\angle GEH = 36$
$m\angle FEG + m\angle F + m\angle FGE = 180$
$m\angle FEG + 110 + 36 = 180$
$m\angle FEG = 180 - 146 = 34°$

44. $m\angle FGH = m\angle FGE + m\angle EGH$
$= m\angle GEH + m\angle FEG$
$= 36 + 34 = 70°$

4-5 TRIANGLE CONGRUENCE: ASA, AAS, AND HL, PAGES 252–259

CHECK IT OUT!

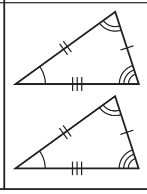
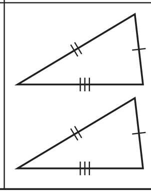
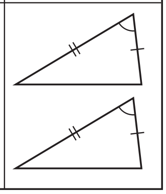
1. Yes; the △ is uniquely determined by AAS.
2. By the Alt. Int. ∠s Thm., $\angle KLN \cong \angle MNL$. $\overline{LN} \cong \overline{NL}$ by the Reflex. Prop. of ≅. No other congruence relationships can be determined, so ASA cannot be applied.
3. 

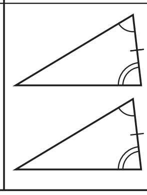
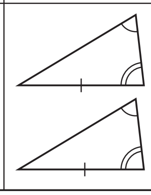
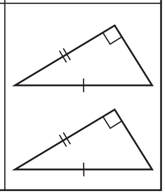
4. Yes; it is given that $\overline{AC} \cong \overline{DB}$. $\overline{CB} \cong \overline{BC}$ by the Reflex. Prop. of ≅. Since $\angle ABC$ and $\angle DCB$ are rt. ∠s, $\triangle ABC \cong \triangle DCB$ by HL.

THINK AND DISCUSS

1. No; the ≅ sides are not corr. sides.
2. Possible answer: corr. ∠s and sides

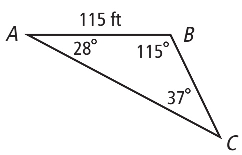
3.

	Def. of △ ≅	SSS	SAS
Words	All 6 corr. parts of 2 △s are ≅.	3 sides of 1 △ are ≅ to 3 sides of another △.	2 sides and an included ∠ of 1 △ are ≅ to 2 sides and an included ∠ in another △.
Pictures			

	ASA	AAS	HL
Words	2 ∠s and an included side of 1 △ are ≅ to 2 ∠s and included side in another △.	2 ∠s and a side of 1 △ are ≅ to their corr. parts in another △.	A leg and hyp. of 1 rt. △ are ≅ to a leg and hyp. in another rt △.
Pictures			

EXERCISES

GUIDED PRACTICE

1. The included side $\overline{BC}$ is enclosed between $\angle ABC$ and $\angle ACB$.
2. 

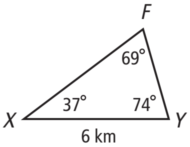
3. Yes; the △ is determined by AAS.
4. Yes; by the Def. of ∠ bisector, $\angle TSV \cong \angle RSV$ and $\angle TVS \cong \angle RVS$. $\overline{SV} \cong \overline{SV}$ by the Reflex. Prop. of ≅. So $\triangle VRS \cong \triangle VTS$ by ASA.
5. No; you need to know that a pair of corr. sides are ≅.

6a. $\overline{QS} \cong \overline{SQ}$ **b.** $\angle RQS \cong \angle PSQ$
c. Rt. ∠ ≅ Thm. **d.** AAS

7. Yes; it is given that $\angle D$ and $\angle B$ are rt. ∠s and $\overline{AD} \cong \overline{BC}$. $\triangle ABC$ and $\triangle CDA$ are rt. △s by def. $\overline{AC} \cong \overline{CA}$ by the Reflex. Prop. of ≅. So $\triangle ABC \cong \triangle CDA$ by HL.
8. No; you need to know that $\overline{VX} \cong \overline{VZ}$.

PRACTICE AND PROBLEM SOLVING

9.



10. Yes; the $\triangle$ is uniquely determined by ASA.

11. No; you need to know that $\angle MKJ \cong \angle MKL$.

12. Yes; by the Alt. Int. $\angle$s Thm., $\angle SRT \cong \angle UTR$, and $\angle STR \cong \angle URT$. $\overline{RT} \cong \overline{TR}$ by the Reflex. Prop. of $\cong$. So $\triangle RST \cong \triangle TUR$ by ASA.

13a. $\angle A \cong \angle D$ **b.** Given

c. $\angle C \cong \angle F$ **d.** AAS

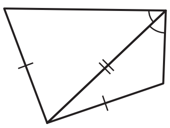
14. No; you need to know that $\angle K$ and $\angle H$ are rt. $\angle$s.

15. Yes; E is a mdpt. So by def., $\overline{BE} \cong \overline{CE}$, and $\overline{AE} \cong \overline{DE}$. $\angle A$ and $\angle D$ are $\cong$ by the Rt. $\angle \cong$ Thm. By def., $\triangle ABE$ and $\triangle DCE$ are rt. $\triangle$s. So $\triangle ABE \cong \triangle DCE$ by HL.

16. AAS proves $\triangle ADB \cong \triangle CDB$; reflection

17. $\triangle FEG \cong \triangle QSR$; rotation

18.

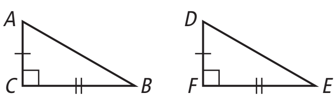


19a. No; there is not enough information given to use any of the congruence theorems.

b. HL can be used, since also $\overline{JL} \cong \overline{JL}$.

20. Proof B is incorrect. The corr. sides are not in the correct order.

21.



It is given that $\triangle ABC$ and $\triangle DEF$ are rt. $\triangle$s.
$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$, and $\angle C$ and $\angle F$ are rt. $\angle$s.
$\angle C \cong \angle F$ by the Rt. $\angle \cong$ Thm.
Thus $\triangle ABC \cong \triangle DEF$ by SAS.

22.

Statements	Reasons
1. $\overline{AD} \parallel \overline{BC}$	1. Given
2. $\angle DAE \cong \angle BCE$	2. Alt. Int. $\angle$s Thm.
3. $\angle AED \cong \angle CEB$	3. Vert. $\angle$s Thm.
4. $\overline{AD} \cong \overline{CB}$	4. Given
5. $\triangle AED \cong \triangle CEB$	5. AAS Steps 2, 3, 4

23.

Statements	Reasons
1. $\overline{KM} \perp \overline{JL}$	1. Given
2. $\angle JKM$ and $\angle LKM$ are rt. $\angle$s	2. Def. of $\perp$
3. $\angle JKM \cong \angle LKM$	3. Rt. $\angle \cong$ Thm.
4. $\overline{JM} \cong \overline{LM}$, $\angle JMK \cong \angle LMK$	4. Given
5. $\triangle JKM \cong \triangle LKM$	5. AAS Steps 3, 4

24. Since 2 sides and the included $\angle$ are equal in measure and therefore $\cong$, you could prove the $\triangle$s $\cong$ using SAS. You could also use HL since the $\triangle$s are rt. $\triangle$s.

25. Check students' constructions.

TEST PREP

26. A
Need $\angle XVZ \cong \angle XWY$ for ASA.

27. J
From figure, 2 corr. side pairs and included $\angle$ pair are $\cong$, i.e., SAS.

28. C
Alt. Int. $\angle$s Thm. gives two $\cong$ $\angle$ pairs, and one non-included $\cong$ side pair is given. AAS proves $\triangle AED \cong \triangle CEB$.

29. G
For AAS, need $\overline{RT} \cong \overline{UW}$. So:

$$\begin{aligned} RT &= UW \\ 6y - 2 &= 2y + 7 \\ 4y &= 9 \\ y &= 2.25 \end{aligned}$$

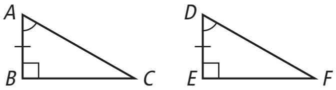
30. No; check students' drawings and constructions; since the lengths of the corr. sides of the 2 $\triangle$s are not equal, the 2 $\triangle$s are not $\cong$ even if the corr. $\angle$s have the same measure.

CHALLENGE AND EXTEND

31. Yes; the sum of the $\angle$ measures in each $\triangle$ must be 180°, which makes it possible to solve for x and y. The value of x is 15, and the value of y is 12. Each $\triangle$ has $\angle$s measuring 82°, 68°, and 30°. $\overline{VU} \cong \overline{VU}$ by the Reflex. Prop. of $\cong$. So $\triangle VSU \cong \triangle VTU$ by ASA or AAS.

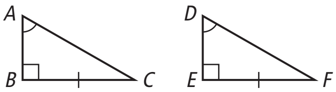
32.

Statements	Reasons
1. $\triangle ABC$ is equil.	1. Given
2. $\overline{AC} \cong \overline{BC}$	2. Def. of equil. $\triangle$
3. C is mdpt. of DE.	3. Given
4. $\overline{DC} \cong \overline{EC}$	4. Def. of mdpt.
5. $\angle DAC$ and $\angle EBC$ are $\cong$. and supp.	5. Given
6. $\angle DAC$ and $\angle EBC$ are rt. $\angle$s.	6. $\angle$s that are $\cong$ and supp. are rt. $\angle$s.
7. $\triangle DAC$ and $\triangle EBC$ are rt. $\triangle$s.	7. Def. of rt. $\triangle$
8. $\triangle DAC \cong \triangle EBC$	8. HL Steps 4, 2

33. 

Case 1: Given rt. $\triangle ABC$ and rt. $\triangle DEF$ with $\angle A \cong \angle D$ and $\overline{AB} \cong \overline{DE}$

Statements	Reasons
1. $\angle A \cong \angle D$	1. Given
2. $\overline{AB} \cong \overline{DE}$	2. Given
3. $\angle B \cong \angle E$	3. Rt. $\angle \cong$ Thm.
4. $\triangle ABC \cong \triangle DEF$	4. ASA *Steps 1, 2, 3*



Case 2; given rt. $\triangle ABC$ and rt. $\triangle DEF$ with $\angle A \cong \angle D$ and $\overline{BC} \cong \overline{EF}$

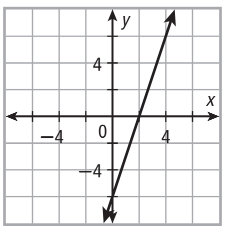
Statements	Reasons
1. $\angle A \cong \angle D$	1. Given
2. $\overline{BC} \cong \overline{EF}$	2. Given
3. $\angle B \cong \angle E$	3. Rt. $\angle \cong$ Thm.
4. $\triangle ABC \cong \triangle DEF$	4. ASA *Steps 1, 3, 2*

34. Third $\angle$s Thm.; if the third $\angle$ pair is $\cong$, then the $\triangle$s are also $\cong$ by AAS.

SPIRAL REVIEW

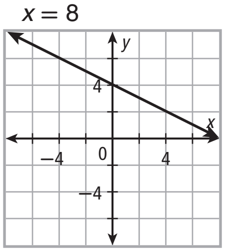
35. x-intercept:
$0 = 3x - 6$
$6 = 3x$
$x = 2$

y-intercept:
$y = 3(0) - 6$
$y = -6$



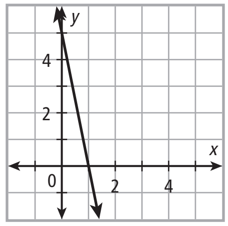
36. x-intercept:
$0 = -\frac{1}{2}x + 4$
$\frac{1}{2}x = 4$
$x = 8$

y-intercept:
$y = -\frac{1}{2}(0) + 4$
$y = 4$



37. x-intercept:
$0 = -5x + 5$
$5x = 5$
$x = 1$

y-intercept:
$y = -5(0) + 5$
$y = 5$



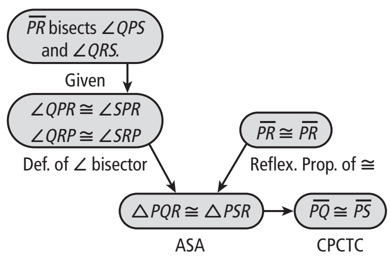
38. $AC = 10$
$x^2 - 6 = 10$
$x^2 = 16$
$x = 4$
(Discard $x = -4$ since $AB > 0$.)
$AB = x + 2 = 4 + 2 = 6$
$BC = x^2 - 2x = 4^2 - 2(4) = 8$

39. $m\angle A + m\angle B + m\angle C = 180$
$53.1 + 90 + m\angle C = 180$
$143.1 + m\angle C = 180$
$m\angle C = 36.9°$

4-6 TRIANGLE CONGRUENCE: CPCTC, PAGES 260–265

CHECK IT OUT!

1. $JL = NL$ and $KL = ML$, so $\overline{JL} \cong \overline{NL}$ and $\overline{KL} \cong \overline{ML}$.
By Vert. $\angle$s Thm., $\angle MLN \cong \angle KLJ$.
By SAS, $\triangle MLN \cong \triangle KLJ$.
By CPCTC, $JK = NM = 41$ ft

2. 

3.

Statements	Reasons
1. J is mdpt. of $\overline{KM}$ and $\overline{NL}$.	1. Given
2. $\overline{KJ} \cong \overline{MJ}$ and $\overline{LJ} \cong \overline{NJ}$	2. Def. of mdpt.
3. $\angle KJL \cong \angle MJN$	3. Vert. $\angle$s Thm.
4. $\triangle KJL \cong \triangle MJN$	4. SAS Steps 2, 3
5. $\angle LKJ \cong \angle NMJ$ or $\angle JLK \cong \angle JNM$	5. CPCTC
6. $\overline{KL} \parallel \overline{MN}$	6. Conv. of Alt. Int. $\angle$s Thm.

4. Use Distance Formula to find side lengths.

$JK = \sqrt{(2-(-1))^2 + ((-1)-(-2))^2}$
$= \sqrt{9+1} = \sqrt{10}$

$KL = \sqrt{((-2)-2)^2 + (0-(-1))^2}$
$= \sqrt{16+1} = \sqrt{17}$

$JL = \sqrt{((-2)-(-1))^2 + (0-(-2))^2}$
$= \sqrt{1+4} = \sqrt{5}$

$RS = \sqrt{(5-2)^2 + (2-3)^2}$
$= \sqrt{9+1} = \sqrt{10}$

$ST = \sqrt{(1-5)^2 + (1-2)^2}$
$= \sqrt{16+1} = \sqrt{17}$

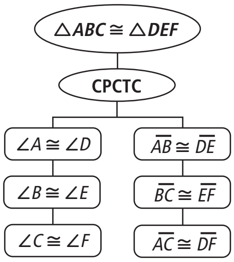
$RT = \sqrt{(1-2)^2 + (1-3)^2}$
$= \sqrt{1+4} = \sqrt{5}$

So $\overline{JK} \cong \overline{RS}$, $\overline{KL} \cong \overline{ST}$, and $\overline{JL} \cong \overline{RT}$. Therefore, $\triangle JKL \cong \triangle RST$ by *SSS*, and $\angle JKL \cong \angle RST$ by CPCTC.

THINK AND DISCUSS

1. *SAS*; $\overline{UW} \cong \overline{XZ}$; $\angle U \cong \angle X$; $\angle W \cong \angle Z$

2.



EXERCISES

GUIDED PRACTICE

1. Corr. $\measuredangle$ and corr. sides

2. $\angle BCA \cong \angle DCE$ by Vert. $\measuredangle$ Thm, $\angle CBA \cong \angle CDE$ *by Rt.* $\angle \cong$ Thm., and $\overline{BC} \cong \overline{DC}$ (given). Therefore $\triangle ABC \cong \triangle EDC$ by ASA. By CPCTC, $\overline{AB} \cong \overline{DE}$, so $AB = DE = 6.3$ m.

3a. Def. of $\perp$

b. Rt. $\angle \cong$ Thm.

c. Reflex. Prop. of $\cong$

d. Def. of mdpt.

e. $\triangle RXS \cong \triangle RXT$

f. CPCTC

4.

Statements	Reasons
1. $\overline{AC} \cong \overline{AD}$, $\overline{CB} \cong \overline{DB}$	1. Given
2. $\overline{AB} \cong \overline{AB}$	2. Reflex. Prop. of $\cong$
3. $\triangle ACB \cong \triangle ADB$	3. SSS Steps 1, 2
4. $\angle CAB \cong \angle DAB$	4. CPCTC
5. $\overline{AB}$ bisects $\angle CAD$.	5. Def. of $\angle$ bisector

5. Use Distance Formula to find side lengths.

$EF = \sqrt{((-1)-(-3))^2 + (3-3)^2}$
$= \sqrt{4+0} = 2$

$FG = \sqrt{((-2)-(-3))^2 + (0-3)^2}$
$= \sqrt{1+9} = \sqrt{10}$

$EG = \sqrt{((-2)-(-1))^2 + (0-3)^2}$
$= \sqrt{1+9} = \sqrt{10}$

$JK = \sqrt{(0-2)^2 + ((-1)-(-1))^2}$
$= \sqrt{4+0} = 2$

$KL = \sqrt{(1-2)^2 + (2-(-1))^2}$
$= \sqrt{1+9} = \sqrt{10}$

$JL = \sqrt{(1-0)^2 + (2-(-1))^2}$
$= \sqrt{1+9} = \sqrt{10}$

So $\overline{EF} \cong \overline{JK}$, $\overline{FG} \cong \overline{KL}$, *and* $\overline{EG} \cong \overline{JL}$. *Therefore* $\triangle EFG \cong \triangle JKL$ *by SSS, and* $\angle EFG \cong \angle JKL$ by CPCTC.

6. Use Distance Formula to find side lengths.

$AB = \sqrt{(4-2)^2 + (1-3)^2}$
$= \sqrt{4+4} = 2\sqrt{2}$

$BC = \sqrt{(1-4)^2 + ((-1)-1)^2}$
$= \sqrt{9+4} = \sqrt{13}$

$AC = \sqrt{(1-2)^2 + ((-1)-3)^2}$
$= \sqrt{1+16} = \sqrt{17}$

$RS = \sqrt{((-3)-(-1))^2 + ((-2)-0)^2}$
$= \sqrt{4+4} = 2\sqrt{2}$

$ST = \sqrt{(0-(-3))^2 + ((-4)-(-2))^2}$
$= \sqrt{9+4} = \sqrt{13}$

$RT = \sqrt{(0-(-1))^2 + ((-4)-0)^2}$
$= \sqrt{1+16} = \sqrt{17}$

So $\overline{AB} \cong \overline{RS}$, $\overline{BC} \cong \overline{ST}$, and $\overline{AC} \cong \overline{RT}$. Therefore $\triangle ABC \cong \triangle RST$ by *SSS*, and $\angle ACB \cong \angle RTS$ by CPCTC.

PRACTICE AND PROBLEM SOLVING

7. $\angle ABC \cong \angle EDC$ by Rt. $\angle \cong$ Thm., $\angle ACB \cong \angle ECD$ by *Vert.* $\measuredangle$ Thm., and $\overline{BC} \cong \overline{DC}$. So $\triangle ABC \cong \triangle EDC$ by ASA. By CPCTC, $AB = DE = 420$ ft.

8.

Statements	Reasons
1. M is mdpt. of $\overline{PQ}$ and $\overline{RS}$.	1. Given
2. $\overline{PM} \cong \overline{QM}$, $\overline{RM} \cong \overline{SM}$	2. Def. of mdpt.
3. $\angle PMS \cong \angle QMR$	3. Vert. $\measuredangle$ Thm.
4. $\triangle PMS \cong \triangle QMR$	4. SAS Steps 2, 3
5. $\overline{QR} \cong \overline{PS}$	5. CPCTC

9.

Statements	Reasons
1. $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$	1. Given
2. $\overline{ZX} \cong \overline{XZ}$	2. Reflex. Prop. of $\cong$
3. $\triangle WXZ \cong \triangle YZX$	3. SSS, steps 1, 2
4. $\angle W \cong \angle Y$	4. CPCTC

10.

Statements	Reasons
1. G is mdpt. of $\overline{FH}$.	1. Given
2. $FG = HG$	2. Def. of mdpt.
3. $\overline{FG} \cong \overline{HG}$	3. Def. of $\cong$.
4. Draw $\overline{EG}$.	4. Exactly 1 line through any 2 pts.
5. $\overline{EG} \cong \overline{EG}$	5. Reflex. Prop. of $\cong$
6. $\overline{EF} \cong \overline{EH}$	6. Given
7. $\triangle EGF \cong \triangle EGH$	7. SSS Steps 3, 5, 6
8. $\angle EFG \cong \angle EHG$	8. CPCTC
9. $\angle 1 \cong \angle 2$	9. $\cong$ Supp. Thm.

11.

Statements	Reasons
1. $\overline{LM}$ bisects $\angle JLK$.	1. Given
2. $\angle JLM \cong \angle KLM$	2. Def. of $\angle$ bisector
3. $\overline{JL} \cong \overline{KL}$	3. Given
4. $\overline{LM} \cong \overline{LM}$	4. Reflex. Prop. of $\cong$
5. $\triangle JLM \cong \triangle KLM$	5. SAS Steps 3, 2, 4
6. $\overline{JM} \cong \overline{KM}$	6. CPCTC
7. M is mdpt. of $\overline{JK}$.	7. Def. of mdpt.

12. $RS = \sqrt{(2-0)^2 + (4-0)^2}$
$= \sqrt{4+16} = 2\sqrt{5}$
$ST = \sqrt{((-1)-2)^2 + (4-3)^2}$
$= \sqrt{9+1} = \sqrt{10}$
$RT = \sqrt{((-1)-0)^2 + (3-0)^2}$
$= \sqrt{1+9} = \sqrt{10}$
$UV = \sqrt{((-3)-(-1))^2 + ((-4)-0)^2}$
$= \sqrt{4+16} = 2\sqrt{5}$
$VW = \sqrt{((-4)-(-3))^2 + ((-1)-(-4))^2}$
$= \sqrt{1+9} = \sqrt{10}$
$UW = \sqrt{((-4)-(-1))^2 + ((-1)-0)^2}$
$= \sqrt{9+1} = \sqrt{10}$
So $\overline{RS} \cong \overline{UV}$, $\overline{ST} \cong \overline{VW}$, and $\overline{RT} \cong \overline{UW}$. Therefore, $\triangle RST \cong \triangle UVW$ by SSS, and $\angle RST \cong \angle UVW$ by CPCTC.

13. $AB = \sqrt{(2-(-1))^2 + (3-1)^2}$
$= \sqrt{9+4} = \sqrt{13}$
$BC = \sqrt{(2-2)^2 + ((-2)-3)^2}$
$= \sqrt{0+25} = 5$
$AC = \sqrt{(2-(-1))^2 + ((-2)-1)^2}$
$= \sqrt{9+9} = 3\sqrt{2}$
$DE = \sqrt{((-1)-2)^2 + ((-5)-(-3))^2}$
$= \sqrt{9+4} = \sqrt{13}$
$EF = \sqrt{((-1)-(-1))^2 + (0-(-5))^2}$
$= \sqrt{0+25} = 5$
$DF = \sqrt{((-1)-2)^2 + (0-(-3))^2}$
$= \sqrt{9+9} = 3\sqrt{2}$
So $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{DF}$. Therefore, $\triangle ABC \cong \triangle DEF$ by SSS, and $\angle BAC \cong \angle EDF$ by CPCTC.

14.

Statements	Reasons
1. $\triangle QRS$ is adj. to $\triangle QTS$. $\overline{QS}$ bisects $\angle RQT$. $\angle R \cong \angle T$.	1. Given
2. $\angle RQS \cong \angle TQS$	2. Def. of $\angle$ bisector
3. $\overline{QS} \cong \overline{QS}$	3. Reflex. Prop. of $\cong$
4. $\triangle RSQ \cong \triangle TSQ$	4. AAS Steps 1, 2, 3
5. $\overline{RS} \cong \overline{TS}$	5. CPCTC
6. $\overline{QS}$ bisects $\overline{RT}$.	6. Def. of bisector

15.

Statements	Reasons
1. E is the mdpt. of $\overline{AC}$ and $\overline{BD}$.	1. Given
2. $\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$	2. Def. of mdpt.
3. $\angle AEB \cong \angle CED$	3. Vert $\angle$s Thm.
4. $\triangle AEB \cong \triangle CED$	4. SAS Steps 2, 3
5. $\angle A \cong \angle C$	5. CPCTC
6. $\overline{AB} \parallel \overline{CD}$	6. Conv. of Alt. Int. $\angle$s Thm.

16a. $\angle ADB$, $\angle ADC$ are rt. $\angle$s, hyp. lengths are =, corr. leg lengths are =. So HL proves $\triangle ADB \cong \triangle ADC$.

b.

Statements	Reasons
1. $\overline{AD} \perp \overline{BC}$	1. Given
2. $\angle ADB$ and $\angle ADC$ are rt. $\angle$s.	2. Def. of $\perp$
3. $\triangle ADB$ and $\triangle ADC$ are rt. $\triangle$s	3. Def. of rt. $\triangle$
4. $AB = AC = 20$ in.	4. Given
5. $\overline{AB} \cong \overline{AC}$	5. Def. of $\cong$
6. $\overline{AD} \cong \overline{AD}$	6. Reflex. Prop. of $\cong$
7. $\triangle ADB \cong \triangle ADC$	7. HL Steps 5, 6
8. $\overline{BD} \cong \overline{CD}$	8. CPCTC

c. $BD^2 + AD^2 = AB^2$
$BD^2 + 10^2 = 20^2$
$BD = \sqrt{400 - 100}$
≈ 17.3 in.
$BC = 2BD \approx 34.6$ in.

17. $\triangle$s are $\cong$ by SAS.
$x + 11 = 2x - 3$
$14 = x$

18. $\triangle$s are $\cong$ by ASA.
$4x + 1 = 6x - 41$
$42 = 2x$
$x = 21$

19.

Statements	Reasons
1. $PS = RQ$	1. Given
2. $\overline{PS} \cong \overline{RQ}$	2. Def. of $\cong$
3. $m\angle 1 = m\angle 4$	3. Given
4. $\angle 1 \cong \angle 4$	4. Def. of $\cong$
5. $\overline{SQ} \cong \overline{QS}$	5. Reflex. Prop. of $\cong$
6. $\triangle PSQ \cong \triangle RQS$	6. SAS Steps 2, 4, 5
7. $\angle 3 \cong \angle 2$	7. CPCTC
8. $m\angle 3 = m\angle 2$	8. Def. of $\cong$

20.

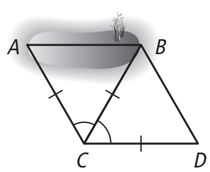
Statements	Reasons
1. $m\angle 1 = m\angle 2$, $m\angle 3 = m\angle 4$	1. Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. Def. of $\cong$
3. $\overline{SQ} \cong \overline{SQ}$	3. Reflex. Prop. of $\cong$
4. $\triangle PSQ \cong \triangle RSQ$	4. ASA Steps 2, 3
5. $\overline{PS} \cong \overline{RS}$	5. CPCTC
6. $PS = RS$	6. Def. of $\cong$

21.

Statements	Reasons
1. $PS = RQ$, $PQ = RS$	1. Given
2. $\overline{PS} \cong \overline{RQ}$, $\overline{PQ} \cong \overline{RS}$	2. Def. of $\cong$
3. $\overline{SQ} \cong \overline{QS}$	3. Reflex. Prop. of $\cong$
4. $\triangle PSQ \cong \triangle RQS$	4. SSS Steps 2, 3
5. $\angle 3 \cong \angle 2$	5. CPCTC
6. $\overline{PQ} \parallel \overline{RS}$	6. Conv. of Alt. Int. $\angle$s Thm.

22. Yes; $\triangle JKM \cong \triangle LMK$ by SSS, so $\angle JKM \cong \angle LMK$ by CPCTC. Therefore, $\overline{JK} \parallel \overline{ML}$ by Conv. of Alt. Int. $\angle$s Thm.

23.



The segs. $\overline{CA}$, $\overline{CD}$, and $\overline{CB}$ must be $\cong$. $\angle ACB \cong \angle DCB$. If $\triangle ACB \cong \triangle DCB$ by SAS, then $AB = DB$.

TEST PREP

24. C
Only way to get a second $\angle$ pair $\cong$ is first to prove $\triangle$s are $\cong$ and then to use CPCTC. But you would use CPCTC to prove $\overline{AC} \cong \overline{AD}$ directly.

25. G
$LNK \cong NLM$, so by CPCTC $\angle LNK \cong \angle NLM$.

26. C
$6x = x + \frac{5}{2}$
$5x = \frac{5}{2}$
$x = \frac{1}{2}$

$10x + y = 40$
$y = 40 - 10x$
$= 40 - 10 \cdot \frac{1}{2}$
$= 35$

27. G
Only corr. parts are ever used. $\cong$ $\triangle$s, $\parallel$ lines, $\perp$ lines all are used.

28. B
$RS = \sqrt{(3-2)^2 + (3-6)^2} = \sqrt{10}$
$ST = \sqrt{(2-6)^2 + (6-6)^2} = 4$
$RT = \sqrt{(6-3)^2 + (6-3)^2} = 3\sqrt{2}$
These lengths only match the $\triangle$ coordinates in B.

CHALLENGE AND EXTEND

29. Any diagonal on any face of the cube is the hyp. of a rt. $\triangle$ whose legs are edges of the cube. Any 2 of these $\triangle$s are $\cong$ by SAS (or LL). Therefore, any 2 diagonals are $\cong$ by CPCTC.

30.

Statements	Reasons
1. Draw $\overline{MK}$.	1. Through any 2 pts. there is exactly 1 line.
2. $\overline{MK} \cong \overline{KM}$	2. Reflex. Prop. of $\cong$
3. $\overline{JK} \cong \overline{LM}$, $\overline{JM} \cong \overline{LK}$	3. Given
4. $\triangle JKM \cong \triangle LMK$	4. SSS Steps 2, 3
5. $\angle J \cong \angle L$	6. CPCTC

31.

Statements	Reasons
1. R is the mdpt. of $\overline{AB}$.	1. Given
2. $\overline{AR} \cong \overline{BR}$	2. Def. of mdpt.
3. $\overline{RS} \perp \overline{AB}$	3. Given
4. $\angle ARS$ and $\angle BRS$ are rt. $\angle$s	4. Def. of $\perp$
5. $\angle ARS \cong \angle BRS$	5. Rt. $\angle \cong$ Thm.
6. $\overline{RS} \cong \overline{RS}$	6. Reflex. Prop. of $\cong$
7. $\triangle ARS \cong \triangle BRS$	7. SAS Steps 2, 5, 6
8. $\overline{AS} \cong \overline{BS}$	8. CPCTC
9. $\angle ASD \cong \angle BSC$	9. Given
10. S is the mdpt. of $\overline{DC}$.	10. Given
11. $\overline{DS} = \overline{CS}$	11. Def. of mdpt.
12. $\triangle ASD \cong \triangle BSC$	12. SAS Steps 8, 9, 11

32. $\angle A \cong \angle E$ (given), $\angle B$ and $\angle D$ are rt. ∡ (from figure), and $BC \cong CD$ (from figure). Therefore, $\triangle ABC \cong \triangle EDC$ by HL. By CPCTC, $AB = DE$.
By Pythag. Thm.,
$$CD^2 + DE^2 = CE^2$$
$$DE^2 = 21^2 - 10^2$$
$$AB = DE = \sqrt{441 - 100} \approx 18 \text{ ft}$$

SPIRAL REVIEW

33. $x = \dfrac{\Sigma x}{n}$
$90 = 90 + 84 + 93 + 88 + 91 + x/6$
$x = 6(90) - (90 + 84 + 93 + 88 + 91) = 94$

34. $P_1 = 3.95 + 0.08m$
$P_1(75) = 3.95 + 0.08(75) = 9.95$
$P_2 = 0.10 \cdot \min(m, 50) + 0.15 \cdot \max(m - 50, 0)$
$P_2(75) = 0.10(50) + 0.15(75 - 50) = 8.75$
The second plan is cheaper.

35. reflection across the x-axis

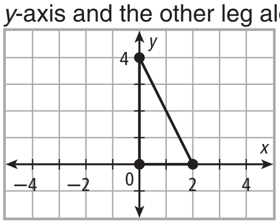
36. translation $(x, y) \rightarrow (x - 3, y - 4)$

37. Yes; it is given that $\angle B \cong \angle D$ and $\overline{BC} \cong \overline{DC}$. By Vert. ∠ Thm., $\angle BCA \cong \angle DCE$. Therefore, $\triangle ABC \cong \triangle EDC$ by ASA.

4-7 INTRODUCTION TO COORDINATE PROOF, PAGES 267–272

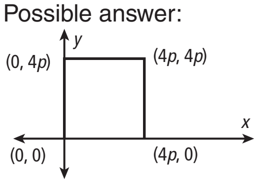
CHECK IT OUT!

1. You can place the longer leg along the y-axis and the other leg along the x-axis.



2. **Proof:**
$\triangle ABC$ is a rt. △ with height AB and base BC.
The area of $\triangle ABC = \frac{1}{2}bh$
$= \frac{1}{2}(4)(6) = 12$ square units
By Mdpt. Formula, coordinates of
$D = \left(\frac{0+4}{2}, \frac{6+0}{2}\right) = (2, 3)$. The x-coord. of D is height of $\triangle ADB$, and base is 6 units.
The area of $\triangle ADB = \frac{1}{2}bh$
$= \frac{1}{2}(2)(6) = 6$ square units
Since $6 = \frac{1}{2}(12)$, area of $\triangle ADB$ is $\frac{1}{2}$ area of $\triangle ABC$.

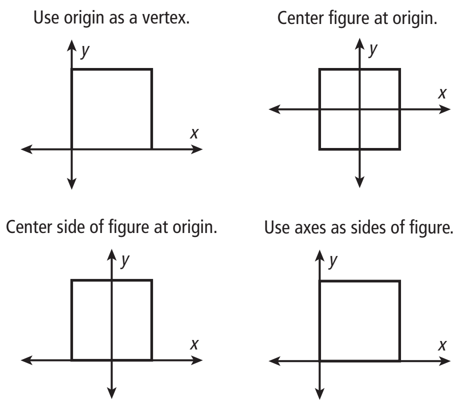
3. Possible answer:



4. $\triangle ABC$ is a rt. △ with height $2j$ and base $2n$.
The area of $\triangle ABC = \frac{1}{2}bh$
$= \frac{1}{2}(2n)(2j) = 2nj$ square units
By the Mdpt. Forumla, the coords. of D are (n, j). The base of $\triangle ABC$ is $2j$ units and the height is n units.
So the area of $\triangle ADB = \frac{1}{2}bh$
$= \frac{1}{2}(2j)(n) = nj$ square units
Since $nj = \frac{1}{2}(2nj)$, the area of $\triangle ADB$ is $\frac{1}{2}$ the area of $\triangle ABC$.

THINK AND DISCUSS

1. Possible answer: By using variables, your results are not limited to specific numerical values.
2. Possible answer: The way you position the figure will affect the coords. assigned to the vertices and therefore, your calculations.
3. Possible answer: If you need to calculate the coords. of a mdpt., assigning $2p$ allows you to avoid using fractions.
4.

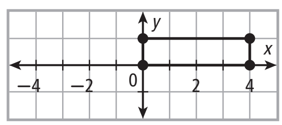


EXERCISES

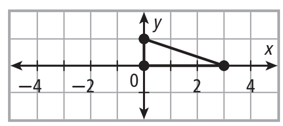
GUIDED PRACTICE

1. Possible answer: In coordinate geometry, a coord. proof is one in which you position figures in the coord. plane to prove a result.

2.



3.

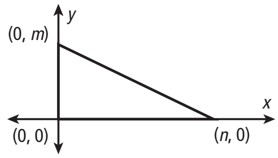


4. By the Mdpt. Forumla, the coords of A are (0, 3) and the coords. of B are (4, 0).
By the Dist. Formula,

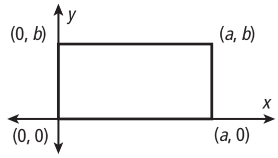
$$PQ = \sqrt{(0-8)^2 + (6-0)^2}$$
$$= \sqrt{(-8)^2 + 6^2} = \sqrt{64 + 36}$$
$$= \sqrt{100} = 10 \text{ units.}$$
$$AB = \sqrt{(0-4)^2 + (3-0)^2}$$
$$= \sqrt{(-4)^2 + 3^2} = \sqrt{16 + 9}$$
$$= \sqrt{25} = 5 \text{ units.}$$

So $AB = \frac{1}{2}PQ$.

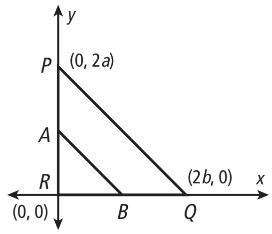
5. Possible answer:



6. Possible answer:



7.



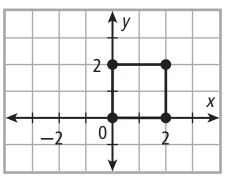
By the Mdpt. Formula, the coords. of A are (0, a) and the coords of B are (b, 0).
By the Dist. Formula,

$$PQ = \sqrt{(0-2b)^2 + (2a)^2} \quad AB = \sqrt{(0-b)^2 + (a-0)^2}$$
$$= \sqrt{(-2b)^2 + (2a)^2} \qquad = \sqrt{(-b)^2 + a^2}$$
$$= \sqrt{4b^2 + 4a^2} \qquad = \sqrt{b^2 + a^2} \text{ units}$$
$$= 2\sqrt{b^2 + a^2} \text{ units}$$

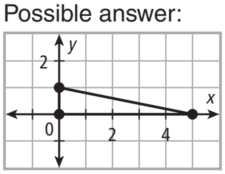
So $AB = \frac{1}{2}PQ$.

PRACTICE AND PROBLEM SOLVING

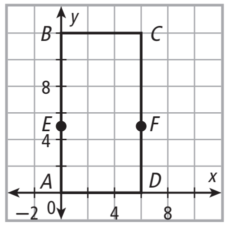
8. Possible answer:



9. Possible answer:



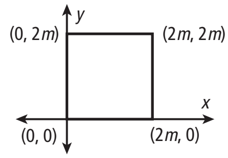
10.



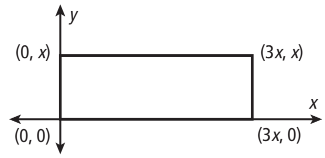
$$E = \left(\frac{0+0}{2}, \frac{0+10}{2}\right) = (0, 5)$$
$$F = \left(\frac{6+6}{2}, \frac{0+10}{2}\right) = (6, 5)$$
$$BC = \sqrt{(6-0)^2 + (10-10)^2} = 6 \text{ units.}$$
$$EF = \sqrt{(6-0)^2 + (5-5)^2} = 6 \text{ units.}$$

So $EF = BC$.

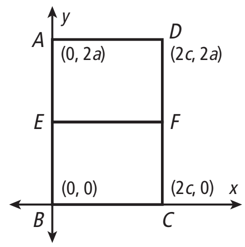
11. Possible answer:



12. Possible answer:



13.



By the Mdpt. Formula, the coords. of E are (0, a) and the coords of F are (2c, a).
By the Dist. Formula,

$$AD = \sqrt{(2c-0)^2 + (2a-2a)^2}$$
$$= \sqrt{(2c)^2} = 2c \text{ units.}$$

Simlarly,

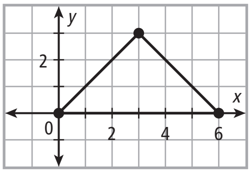
$$EF = \sqrt{(2c-0)^2 + (a-a)^2}$$
$$= \sqrt{(2c)^2} = 2c \textit{ units.}$$

So $EF = AD$.

14. Let endpts. be (x, y) *and* (z, w). By Mdpt. Formula,

$$(0, 0) = \left(\frac{x+z}{2}, \frac{y+w}{2}\right)$$
$$\frac{x+z}{2} = 0$$
$$x + z = 0$$
$$z = -x$$
$$\frac{y+w}{2} = 0$$
$$y + w = 0$$
$$w = -y$$

Endpts are (x, y) and ($-x$, $-y$).

15a. 

b. Total distance $= EW + WC$

$$= \sqrt{(3-0)^2 + (3-0)^2} + \sqrt{(6-3)^2 + (0-3)^2}$$
$$= 3\sqrt{2} + 3\sqrt{2} = 6\sqrt{2} \approx 8.5$$

16. Let $A = (0, 0)$, $B = (a, 0)$, and $C = (0, 2a)$.
Perimeter $= AB + BC + CA$

$$= a + \sqrt{(0-a)^2 + (2a-0)^2} + 2a$$
$$= a\left(3 + \sqrt{5}\right) \text{ units}$$

$\triangle ABC$ has base AB, height AC.

Area $= \frac{1}{2}bh$

$$= \frac{1}{2}(a)(2a) = a^2 \text{ square units}$$

17. Let $A = (0, 0)$, $B = (s, 0)$, $C = (s, t)$, and $D = (0, t)$.
Perimeter $= AB + BC + CD + DA$
$= s + t + s + t = 2s + 2t$ units
Area $= \ell w = st$ square units

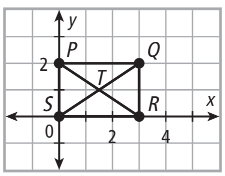
18. (n, n) **19.** $(p, 0)$

20. $\sqrt{(-23.2 - (-25))^2 + (31.4 - 31.5)^2} \approx 1.8$ units
$\sqrt{(-24 - (-23.2))^2 + (31.1 - 31.4)^2} \approx 0.9$ units
$\sqrt{(-24 - (-25))^2 + (31.1 - 31.5)^2} \approx 1.1$ units
1.8 is twice 0.9. The dist. between 2 of the locations is approx. twice the dist. between another 2 locations.

21. $AB = \sqrt{(70 - (-30))^2 + ((-30) - 50)^2}$
≈ 128 nautical miles

Mdpt. of $AB = \left(\frac{-30 + 70}{2}, \frac{50 + (-30)}{2}\right)$
$= (20,10)$

So, P is the mdpt of $\overline{AB}$.

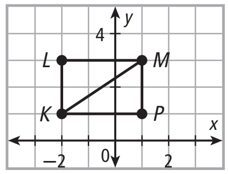
22. 

The area of the rect. is $A = \ell w = 3(2) = 6$ square units. For $\triangle RST$, the base is 3 units, and the height is 1 unit. So the area of
$\triangle RST = \frac{1}{2}bh = \frac{1}{2}(3)(1) = 1.5$ square units.
Since $\frac{1}{4}(6) = 1.5$, the area of $\triangle RST$ is $\frac{1}{4}$ of the area of the rect.

23. By Dist. Formula,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ and}$$
$$AM = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$
$$= \sqrt{\left(\frac{x_1 + x_2}{2} - \frac{2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2}{2} - \frac{2y_1}{2}\right)^2}$$
$$= \sqrt{\frac{1}{4}(x_2 - x_1)^2 + \frac{1}{4}(y_2 - y_1)^2}$$
$$= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

So $AM = \frac{1}{2}AB$.

24. 

Proof: By Dist. Formula,

$$KL = \sqrt{(-2+2)^2 + (1-3)^2} = \sqrt{0+4} = 2$$
$$MP = \sqrt{(1-1)^2 + (3-1)^2} = \sqrt{0+4} = 2$$
$$LM = \sqrt{(-2-1)^2 + (3-3)^2} = \sqrt{9+0} = 3$$
$$PK = \sqrt{(1+2)^2 + (1-1)^2} = \sqrt{9+0} = 3$$

Thus $KL = MP$ and $LM = PK$ by Trans. Prop. of $\cong$. $\overline{KL} \cong \overline{MP}$ and $\overline{LM} \cong \overline{PK}$ by def. of $\cong$, and $\overline{KM} \cong \overline{MK}$ by Reflex. Prop. of $\cong$. Thus $\triangle KLM \cong \triangle MPK$ by SSS.

25. You are assuming the figure has a rt. $\angle$.

26a. $BD = BC + CD$
$= AE + CD$
$= 28 + 10 = 38$ in.
By Dist. Formula,

$$DE = \sqrt{CD^2 + CE^2}$$
$$CE^2 = DE^2 - CD^2$$
$$CE = \sqrt{26^2 - 10^2} = 24 \text{ in.}$$

b. $B = (24, 0)$; $C = (24, 28)$; $D = (24, 38)$; $E = (0, 28)$

TEST PREP

27. B; Mdpt. Formula shows B is true.

28. F; G, H, and J are all possible vertices.

29. D; Perimeter $= a + b + a + b = 2a + 2b$

30. H; $\left(\frac{-1 + 7}{2}, \frac{2 + 8}{2}\right) = (3, 5) = C$

CHALLENGE AND EXTEND

31. $(a + c, b)$

32. $(n + p - n, h - h) = (p, 0)$

33. Possible answer: Rotate $\triangle$ 180° about (0, 0) and translate by $(0, 2s)$. The new coords. would be $(0, 0)$, $(2s, 0)$, $(0, 2s)$.

34. E is intersection of 2 given lines. At E, $y = \frac{g}{f}x$ and $y = -\frac{g}{f}x + 2g$.

$\frac{g}{f}x = -\frac{g}{f}x + 2g$	Set eqns. = to each other.
$2\frac{g}{f}x = 2g$	Combine like terms.
$x = f$	Simplify.
$y = \frac{g}{f}x$	Given
$y = \frac{g}{f}f$	Subst.
$y = g$	Simplify.
$E = (f, g)$	

SPIRAL REVIEW

35. $x = \dfrac{-18 \pm \sqrt{18^2 - 4(8)(-5)}}{2(8)}$

$= \dfrac{-18 \pm 22}{16} = \dfrac{1}{4}$ or $-2\dfrac{1}{4}$

36. $x = \dfrac{-3 \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)}$

$= \dfrac{-3 \pm \sqrt{29}}{2} \approx 1.19$ or -4.19

37. $x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(3)(-10)}}{2(3)}$

$= \dfrac{1 \pm 11}{6} = 2$ or $-1\dfrac{2}{3}$

38. Think:
Use Supp. Int. ∠s Thm.
$x + 68 = 180$
$x = 112$

39. Think:
Use Alt. Int. ∠s Thm.
$2y + 24 = 68$
$2y = 44$
$y = 22$

40. $AB = 3$
$BC = \sqrt{(-3+1)^2 + (1-3)^2} = 2\sqrt{2}$
$AC = \sqrt{(-3+4)^2 + (1-3)^2} = \sqrt{5}$
$ED = 3$
$DF = \sqrt{(2-0)^2 + (-4+2)^2} = 2\sqrt{2}$
$EF = \sqrt{(2-3)^2 + (-4+2)^2} = \sqrt{5}$
Therefore, $\triangle ABC \cong \triangle EDF$ by SSS, and $\angle ABC \cong \angle EDF$ by CPCTC.

4-8 ISOSCELES AND EQUILATRAL TRIANGLES, PAGES 273–279

CHECK IT OUT!

1. 4.2×10^{13}; since there are 6 months between September and March, the ∠ measures will be approx. the same between Earth and the star. By the Conv. of the Isosc. △ Thm., the △s created are isosc., and the dist. is the same.

2a. $m\angle G = m\angle H = x$
$m\angle F + m\angle G + m\angle H = 180$
$48 + x + x = 180$
$2x = 132$
$x = 66$
Thus $m\angle H = x = 66°$.

b. $m\angle N = m\angle P$
$6y = 8y - 16$
$16 = 2y$
$y = 8$
Thus $m\angle N = 6y = 6(8) = 48°$.

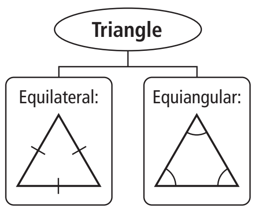
3. $\triangle JKL$ is equilateral.
$4t - 8 = 2t + 1$
$2t = 9$
$t = 4.5$
$JL = 2t + 1$
$= 2(4.5) + 1 = 10$

4. Proof:
By Mdpt. Formula, coords. of X are $\left(\dfrac{-2a+0}{2}, \dfrac{0+2b}{2}\right) = (-a, b)$, coords. of Y are $\left(\dfrac{2a+0}{2}, \dfrac{0+2b}{2}\right) = (a, b)$, and coords of Z are $\left(\dfrac{-2a+2a}{2}, \dfrac{0+0}{2}\right) = (0, 0)$.
By Dist. Formula,
$XZ = \sqrt{(0+a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$, and
$YZ = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$
Since $XZ = YZ$, $\overline{XZ} \cong \overline{YZ}$ by definition. So $\triangle XYZ$ is isosc.

THINK AND DISCUSS

1. An equil. △ is also an equiangular △, so the 3 ∠s have the same measure. They must add up to 180° by the △ Sum Thm. So each ∠ must measure 60°.

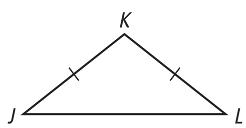
2.



EXERCISES

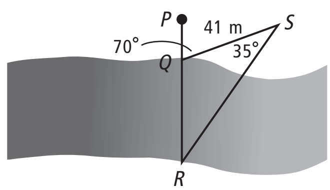
GUIDED PRACTICE

1.



legs: $\overline{KJ}$ and $\overline{KL}$
base: $\overline{JL}$
base ∠s: $\angle J$ and $\angle L$

2.



By the Ext. ∠ Thm., $m\angle R = 35°$. Since $m\angle R = m\angle S$ by the Conv. of the Isosc. △ Thm., $QR = QS = 41$ m.

3. Think: Use Isosc. △ Thm., △ ∠ Sum Thm., and Vert. ∠ Thm.
$m\angle B = m\angle A = 31°$
$m\angle A + m\angle B + m\angle ABC = 180$
$31 + 31 + m\angle ABC = 180$
$m\angle ABC = 118°$
$m\angle ECD = m\angle ABC = 118°$

4. Think: Use Isosc. △ Thm. and △ ∠ Sum Thm.
$m\angle J = m\angle K$
$m\angle J + m\angle K + m\angle L = 180$
$2m\angle K + 82 = 180$
$2m\angle K = 98$
$m\angle K = 49°$

5. Think: Use Isosc. △ Thm.
$m\angle X = m\angle Y$
$5t - 13 = 3t + 3$
$2t = 16$
$t = 8$
$m\angle X = 5t - 13 = 27°$

6. Think: Use Isosc. △ Thm. and △ ∠ Sum Thm.
$m\angle B = m\angle C = 2x$
$m\angle A + m\angle B + m\angle C = 180$
$4x + 2x + 2x = 180$
$8x = 180$
$x = 22.5$
$m\angle A = 4x = 90°$

7. Think: Use Equilat. △ Thm. and △ ∠ Sum Thm.
$\angle R \cong \angle S \cong \angle T$
$m\angle R + m\angle S + m\angle T = 180$
$12y + 12y + 12y = 180$
$36y = 180$
$y = 5$

8. Think: Use Equilat. △ Thm. and △ ∠ Sum Thm.
$\angle L \cong \angle M \cong \angle N$
$m\angle L + m\angle M + m\angle N = 180$
$3(10x + 20) = 180$
$30x = 120$
$x = 4$

9. Think: Use Equiang. △ Thm.
$\overline{AB} \cong \overline{BC} \cong \overline{AC}$
$BC = AC$
$6y + 2 = -y + 23$
$7y = 21$
$y = 3$
$BC = 6y + 2$
$= 6(3) + 2 = 20$

10. Think: Use Equiang. △ Thm.
$\overline{HJ} \cong \overline{JK} \cong \overline{HK}$
$HJ = JK$
$7t + 15 = 10t$
$15 = 3t$
$t = 5$
$JK = 10t$
$= 10(5) = 50$

11. **Proof:**
It is given that △ABC is rt. isosc., $\overline{AB} \cong \overline{BC}$, and X is the mdpt. of $\overline{AC}$. By Mdpt. Formula, coords. of X are $\left(\frac{0 + 2a}{2}, \frac{2a + 0}{2}\right) = (a, a)$. By Dist. Formula,
$AX = \sqrt{(a - 0)^2 + (a - 2a)^2} = a\sqrt{2}$ and
$BX = \sqrt{(a - 0)^2 + (a - a)^2} = a\sqrt{2} = AX$. So △$AXB$ is isosc. by def. of an isosc. △.

PRACTICE AND PROBLEM SOLVING

12. By ∠ Add. Post., $m\angle ATB = 80 - 40 = 40°$. $m\angle BAT = 40°$ by Alt. Int. ∠s Thm. $\angle ATB \cong \angle BAT$ by def. of ≅. Since △ABT is isosc. by Conv. of Isosc. △ Thm., $BT = BA = 2.4$ mi.

13. Think: use Isosc. △ Thm., △ ∠ Sum Thm., and Vert. ∠s Thm.
$m\angle B = m\angle ACB$
$m\angle A + m\angle B + m\angle ACB = 180$
$96 + 2m\angle ACB = 180$
$m\angle ACB = 42°$
$m\angle DCE = m\angle ACB = 42°$
$m\angle D = m\angle E$
$m\angle D + m\angle E + m\angle DCE = 180$
$2m\angle E + 42 = 180$
$m\angle E = 69°$

14. Think: Use Isosc. △ Thm. and △ ∠ Sum Thm.
$m\angle U = m\angle S = 57°$
$m\angle SRU + m\angle S + m\angle U = 180$
$m\angle SRT + m\angle TRU + 57 + 57 = 180$
$2m\angle TRU = 66$
$m\angle TRU = 33°$

15. $m\angle D = m\angle E$
$x^2 = 3x + 10$
$x^2 - 3x - 10 = 0$
$(x - 5)(x + 2) = 0$
$x = 5$ or -2
$m\angle D + m\angle E + m\angle F = 180$
$x^2 + 3x + 10 + m\angle F = 180$
$m\angle F = 180 - 50$ or $180 - 8$
$= 130°$ or $172°$

16. Think: Use Isosc. △ Thm. and △ ∠ Sum Thm.
$m\angle A = m\angle B = (6y + 1)°$
$m\angle A + m\angle B + m\angle C = 180$
$2(6y + 1) + 21y + 3 = 180$
$33y = 165$
$y = 5°$
$m\angle A = 6y + 1 = 31°$

17. Think: Use Equilat. △ Thm. and △ ∠ Sum Thm.
$\angle F \cong \angle G \cong \angle H$
$m\angle F + m\angle G + m\angle H = 180$
$3\left(\frac{z}{2} + 14\right) = 180$
$z + 28 = 120$
$z = 92$

18. Think: Use Equilat. △ Thm. and △ ∠ Sum Thm.
$\angle L \cong \angle M \cong \angle N$
$m\angle L + m\angle M + m\angle N = 180$
$3(1.5y - 12) = 180$
$y - 8 = 40$
$y = 48$

19. Think:
use Equiang. △ Thm.
$\overline{BC} \cong \overline{CD} \cong \overline{BD}$
$BC = CD$
$\frac{3}{2}x + 2 = \frac{5}{4}x + 6$
$6x + 8 = 5x + 24$
$x = 16$
$BC = \frac{3}{2}x + 2$
$= \frac{3}{2}(16) + 2 = 26$

20. Think:
use Equiang. △ Thm.
$\overline{XY} \cong \overline{YZ} \cong \overline{XZ}$
$XY = XZ$
$2x = \frac{5}{2}x - 5$
$5 = \frac{1}{2}x$
$x = 10$
$XZ = XY$
$= 2x$
$= 2(10) = 20$

21. **Proof:** It is given that $\triangle ABC$ is isosc., $\overline{AB} \cong \overline{AC}$, P is mdpt. of $\overline{AB}$, and Q is mdpt. of $\overline{AC}$. By Mdpt. Formula, coords. of P are (a, b), and coords. of Q are $(3a, b)$. By Dist. Formula,
$PC = QB = \sqrt{9a^2 + b^2}$, so $\overline{PC} \cong \overline{QB}$ by def. of ≅.

22. always

23. sometimes

24. sometimes

25. never

26. No; if a base ∠ is obtuse, the other base ∠ must also be obtuse since they are ≅. But the sum of the ∠ measures of the △ cannot be > 180°.

27a. $\overline{PS} \cong \overline{PT}$, so by Isosc. △ Thm.,
$m\angle PTS = m\angle PST = 71°$. By △ ∠ Sum Thm,
$m\angle SPT + m\angle PTS + m\angle PST = 180$
$m\angle SPT + 71 + 71 = 180$
$m\angle SPT = 38°$

b. $\overline{PQ} \cong \overline{PR}$, so by Isosc. △ Thm.,
$m\angle PQR = m\angle PRQ$. By △ ∠ Sum Thm,
$m\angle PQR + m\angle PRQ + m\angle QPR = 180$
$2m\angle PQR + (m\angle QPS + m\angle SPT + m\angle TPR) = 180$
$2m\angle PQR + 18 + 38 + 18 = 180$
$2m\angle PQR = 106$
$m\angle PQR = 53°$
$m\angle PRQ = 53°$

28. Let 3rd ∠ of △ be ∠4.
$m\angle 1 = m\angle 4 = 58°$ (Alt. Int. ∠s Thm., Isosc. △ Thm.)
$m\angle 2 + 58 + 58 = 180$
$m\angle 2 = 64°$
$m\angle 2 + m\angle 3 = 180$ (supp. ∠s)
$58 + m\angle 3 = 180$
$m\angle 3 = 122°$

29. Let 3rd ∠ of left △ be ∠4.
$m\angle 3 = m\angle 4$ (Isosc. △ Thm.)
$m\angle 3 + m\angle 4 + 74 = 180$
$2m\angle 3 = 106$
$m\angle 3 = 53°$
$m\angle 1 + m\angle 4 = 180$ (supp. ∠s)
$m\angle 1 + 53 = 180$
$m\angle 1 = 127°$
Let 3rd ∠ of right △ be ∠5.
$m\angle 2 = m\angle 5$ (Isosc. △ Thm.)
$m\angle 1 + m\angle 2 + m\angle 5 = 180$
$127 + 2m\angle 2 = 180$
$m\angle 2 = 26.5°$

30. **Proof:** It is given that $\triangle ABC$ is isosc., $\overline{BA} \cong \overline{BC}$, and X is the mdpt. of $\overline{AC}$. Assign the coords. $A(0, 2a)$, $B(0, 0)$, and $C(2a, 0)$. By the Mdpt. Formula, coords. of X are (a, a). By Dist. Formula, $AX = XB = XC = a\sqrt{2}$. So $\triangle AXB \cong \triangle CXB$ by SSS.

31. Check students' drawings. The ∠s are approx. 34°, 34°, and 112°. Conjecture should be that △ is isosc. Conjecture is correct since two short sides have equal measure ($\sqrt{65}$ units).

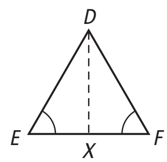
32. List all (unordered) triples of natural numbers such that:
- at least two are equal
- sum of leg lengths > base length
- perimeter is 18

4 △s: (5, 5, 8), (6, 6, 6), (7, 7, 4), (8, 8, 2).

33. In left △: $40 + x + x = 180$
$2x = 140$
$x = 70$
In right △: $x + 2(3y - 5) = 180$
$60 + 6y = 180$
$y = 20$

34. In left △: all ∠s measure 60°.
In right △: obtuse ∠ measures
$180° - 60° = 120°$.
$2(5x + 15) + 120 = 180$
$10x + 150 = 180$
$x = 3$

35.



Statements	Reasons
1. $\triangle DEF$	1. Given
2. Draw the bisector of $\angle EDF$ so that it intersects $\overline{EF}$ at X.	2. Every ∠ has a unique bisector.
3. $\angle EDX \cong \angle FDX$	3. Def. of ∠ bisector
4. $\overline{DX} \cong \overline{DX}$	4. Refl. Prop. of ≅
5. $\angle E \cong \angle F$	5. Given
6. $\triangle EDX \cong \triangle FDX$	6. AAS Steps 3, 5, 4
7. $\overline{DE} \cong \overline{DF}$	7. CPCTC

36a. $\angle B \cong \angle C$

b. Isosc. △ Thm

c. Trans. Prop. of ≅

37. $\triangle DEF$ with $\angle D \cong \angle E \cong \angle F$ is given. Since $\angle E \cong \angle F$, $\overline{DE} \cong \overline{DF}$ by Conv. of Isosc. △ Thm. Similarly, since $\angle D \cong \angle F$, $\overline{EF} \cong \overline{DE}$. By the Trans. Prop. of ≅, $\overline{EF} \cong \overline{DF}$. Combining the ≅ statements, $\overline{DE} \cong \overline{DF} \cong \overline{EF}$, and $\triangle DEF$ is equil. by def.

38. By the Ext. ∠ Thm., $m\angle C = 45°$, so $\angle A \cong \angle C$. $BC = AB$ by the Conv. of the isosc. △ Thm. So the distance to island C is the same as the distance traveled from A to B.

39. 1. $\triangle ABC \cong \angle CBA$ (Given)
2. $\overline{AB} \cong \overline{CB}$ (CPCTC)
3. $\triangle ABC$ (Def. of Isosc. ρ)

40. Two sides of a $\triangle$ are $\cong$ if and only if the ∠s opp. those sides are $\cong$.

41.

Statements	Reasons
1. $\triangle ABC$ and $\triangle DEF$	1. Given
2. Draw $\overrightarrow{EF}$ so that $FG = CB$.	2. Through any 2 pts. there is exactly 1 line.
3. $\overline{FG} \cong \overline{CB}$	3. Def. of $\cong$ segs.
4. $\overline{AC} \cong \overline{DF}$	4. Given
5. $\angle C$, $\angle F$ are rt. ∠s.	5. Given
6. $\overline{DF} \perp \overline{EG}$	6. Def. of $\perp$ lines
7. $\angle DFG$ is rt. $\angle$	7. Def. of rt. $\angle$
8. $\angle DFG \cong \angle C$	8. Rt. $\angle \cong$ Thm.
9. $\triangle ABC \cong \triangle DGF$	9. SAS Steps 3, 8, 4
10. $\overline{DG} \cong \overline{AB}$	10. CPCTC
11. $\overline{AB} \cong \overline{DE}$	11. Given
12. $\overline{DG} \cong \overline{DE}$	12. Trans. Prop. of $\cong$
13. $\angle G \cong \angle E$	13. Isosc. $\triangle$ Thm.
14. $\angle DFG \cong \angle DFE$	14. Rt. $\angle \cong$ Thm.
15. $\triangle DGF \cong \triangle DEF$	15. AAS Steps 13, 14, 12
16. $\triangle ABC \cong \triangle DEF$	16. Trans. Prop. of $\cong$

42. A
$$m\angle VUT = m\angle VTU$$
$$2m\angle VUT + m\angle VTU + m\angle TUV = 180$$
$$2m\angle VUT + 20 = 180$$
$$m\angle VUT = 80°$$
$$m\angle VUR + m\angle VUT = 90$$
$$m\angle VUR + 80 = 90$$
$$m\angle VUR = 10°$$

43. H
$$y + 10 = 3y - 5$$
$$15 = 2y$$
$$y = 7\frac{1}{2}$$

44. 13.5
$$6t - 9 + 4t + 4t = 180$$
$$14t = 189$$
$$t = 13.5$$

CHALLENGE AND EXTEND

45. It is given that $\overline{JK} \cong \overline{JL}$, $\overline{KM} \cong \overline{KL}$, and $m\angle J = x°$. By the $\triangle$ Sum Thm., $m\angle JKL + m\angle JLK + x° = 180°$. By the Isosc. $\triangle$ Thm., $m\angle JKL = m\angle JLK$. So $2(m\angle JLK) + x° = 180°$. or $m\angle JLK = \left(\frac{180 - x}{2}\right)°$. Since $m\angle KML = m\angle JLK$, $m\angle KML = \left(\frac{180 - x}{2}\right)°$ by the Isosc. $\triangle$ Thm. By the $\triangle$ Sum Thm., $m\angle MKL + m\angle JLK + m\angle KML = 180°$ or $m\angle MKL = 180° - \left(\frac{180 - x}{2}\right)° - \left(\frac{180 - x}{2}\right)°$. Simplifying gives $m\angle MKL = x°$.

46. Let $A = (x, y)$.
$$4a^2 = AB^2$$
$$= x^2 + y^2$$
$$= AC^2$$
$$= (x - 2a)^2 + y^2$$
$$= x^2 - 4ax + 4a^2 + y^2$$
$$= 4a^2 - 4ax + 4a^2$$
$$4ax = 4a^2$$
$$x = a$$
$$y = \pm\sqrt{4a^2 - x^2}$$
$$= \pm a\sqrt{3}$$
$$(x, y) = \left(a, a\sqrt{3}\right)$$

47. $(2a, 0)$, $(0, 2b)$, or any pt. on the $\perp$ bisector of $\overline{AB}$.

SPIRAL REVIEW

48. $x^2 + 5x + 4 = 0$
$(x + 4)(x + 1) = 0$
$x = -4$ or -1

49. $x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$
$x = 3$ or 1

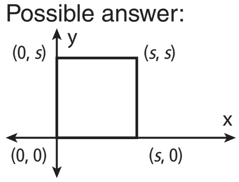
50. $x^2 - 2x + 1 = 0$
$(x - 1)(x - 1) = 0$
$x = 1$

51. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{0 - 2} = \frac{6}{-2} = -3$

52. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - (-10)}{20 - (-5)} = 0$

53. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 7}{10 - 4} = \frac{4}{6} = \frac{2}{3}$

54. Possible answer:



READY TO GO ON? PAGE 281

1. It is given that $\overline{AC} \cong \overline{BC}$, and $\overline{DC} \cong \overline{DC}$ by Reflex. Prop. of $\cong$. By the Rt. $\angle \cong$ Thm., $\angle ACD \cong \angle BCD$. Therefore, $\triangle ACD \cong \triangle BCD$ by SAS.

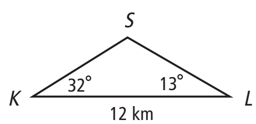
2.

Statements	Reasons
1. $\overline{JK}$ bisects $\angle MJN$.	1. Given
2. $\angle MJK \cong \angle NJK$	2. Def. of $\angle$ bisector
3. $\overline{MJ} \cong \overline{NJ}$	3. Given
4. $\overline{JK} \cong \overline{JK}$	4. Reflex. Prop of $\cong$
5. $\triangle MJK \cong \triangle NJK$	5. SAS Steps 3, 2, 4

3. Yes, since $\overline{SU} \cong \overline{US}$.

4. No; need $\overline{AC} \cong \overline{DB}$.

5.



6. Yes; the $\triangle$ is uniquely determined by ASA.

7.

Statements	Reasons
1. $\overline{CD} \parallel \overline{BE}$ and $\overline{DE} \parallel \overline{CB}$	1. Given
2. $\angle DEC \cong \angle BCE$ and $\angle DCE \cong \angle BEC$	2. Alt. Int. $\angle$s Thm.
3. $\overline{CE} \cong \overline{EC}$	3. Reflex. Prop of $\cong$
4. $\triangle DEC \cong \triangle BCE$	4. ASA Steps 2, 3
5. $\angle D \cong \angle B$	5. CPCTC

8. Check students' drawings; possible answer: vertices at (0, 0), (9, 0), (9, 9), and (0, 9).

9. It is given that $ABCD$ is a rect. M is the mdpt. of $\overline{AB}$, and N is the mdpt. of $\overline{AD}$. Use coords. $A(0, 0)$, $B(2a, 0)$, $C(2a, 2b)$, and $D(0, 2b)$. By Mdpt. Formula, coords. of M are $\left(\frac{0 + 2a}{2}, \frac{0 + 0}{2}\right) = (a, 0)$, and coords. of N are $\left(\frac{0 + 0}{2}, \frac{0 + 2b}{2}\right) = (0, b)$.
Area of rect. $ABCD = \ell w = (2a)(2b) = 4ab$.
Area of $\triangle AMN = \frac{1}{2}bh = \frac{1}{2}ab$, which is $\frac{1}{8}$ the area of rect. $ABCD$.

10. $m\angle E = m\angle D = 2x°$
$m\angle C + m\angle D + m\angle E = 180$
$5x + 2x + 2x = 180$
$9x = 180$
$x = 20$
$m\angle C = 5x = 100°$

11. By Equiang. $\triangle$ Thm.,
$\overline{RS} \cong \overline{RT} \cong \overline{ST}$
$RS = RT$
$2w + 5 = 8 - 4w$
$6w = 3$
$w = 0.5$
$ST = RS = 2(0.5) + 5 = 6$

12. It is given that isosc. $\triangle JKL$ has coords. $J(0, 0)$, $K(2a, 2b)$, and $L(4a, 0)$. M is mdpt. of $\overline{JK}$, and N is mdpt. of $\overline{KL}$. By Mdpt. Formula, coords. of M are $\left(\frac{0 + 2a}{2}, \frac{0 + 2b}{2}\right) = (a, b)$, and coords. of N are $\left(\frac{2a + 4a}{2}, \frac{2b + 0}{2}\right) = (3a, b)$. By Dist. Formula,
$MK = \sqrt{(2a - a)^2 + (2b - b)^2} = \sqrt{a^2 + b^2}$, and
$NK = \sqrt{(2a - 3a)^2 + (2b - b)^2} = \sqrt{a^2 + b^2}$.
Thus $\overline{MK} \cong \overline{NK}$. So $\triangle KMN$ is isosc. by def. of isosc. $\triangle$.

STUDY GUIDE: REVIEW, PAGES 284–287

1. isosceles
2. corresponding angles
3. included side

LESSON 4-1

4. equiangular; equilat.
5. obtuse; scalene

LESSON 4-2

6. Think: Use Ext. $\angle$ Thm.
$m\angle N + m\angle P = m(\text{ext. } \angle Q)$
$y + y = 120$
$y = 60$
$m\angle N = y = 60°$

7. Think: Use $\triangle$ $\angle$ Sum Thm.
$m\angle L + m\angle M + m\angle N = 180$
$8x + 2x + 1 + 6x - 1 = 180$
$16x = 180$
$x = 11.25$
$m\angle N = 6x - 1 = 66.5°$

LESSON 4-3

8. $\overline{PR} \cong \overline{XZ}$
9. $\angle Y \cong \angle Q$

10. $m\angle CAD = m\angle ACB$
$2x - 3 = 47$
$2x = 50$
$x = 25$

11. $CD = AB$
$3y + 1 = 15 - 4y$
$7y = 14$
$y = 2$
$CD = 3y + 1 = 7$

LESSON 4-4

12.

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$, $\overline{DB} \cong \overline{AE}$	1. Given
2. $\overline{DA} \cong \overline{AD}$	2. Reflex. Prop. of $\cong$
3. $\triangle ADB \cong \triangle DAE$	3. SSS Steps 1, 2

13.

Statements	Reasons
1. $\overline{GJ}$ bisects $\overline{FH}$, and $\overline{FH}$ bisects $\overline{GJ}$.	1. Given
2. $\overline{GK} \cong \overline{JK}$, $\overline{FK} \cong \overline{HK}$	2. Def. of seg. bisector
3. $\angle GKF \cong \angle JKH$	3. Vert. $\angle$s Thm.
4. $\triangle FGK \cong \triangle HJK$	4. SAS Steps 2, 3

14. $BC = x^2 + 36 = (-6)^2 + 36 = 72$
$YZ = 2x^2 = 2(-6)^2 = 72 = BC$
$\overline{BC} \cong \overline{YZ}$; $\angle C \cong \angle Z$; $\overline{AC} \cong \overline{XZ}$. So $\triangle ABC \cong \triangle XYZ$ by SAS.

15. $PQ = y - 1 = 25 - 1 = 24$
$QR = y = 25$
$PR = y^2 - (y - 1)^2 - 42 = (25)^2 - (24)^2 - 42 = 7$
$\overline{LM} \cong \overline{PQ}$; $\overline{MN} \cong \overline{QR}$; $\overline{LN} \cong \overline{PR}$.
So $\triangle LMN \cong \triangle PQR$ by SSS.

LESSON 4-5

16.

Statements	Reasons
1. C is mdpt. of $\overline{AG}$.	1. Given
2. $\overline{GC} \cong \overline{AC}$	2. Def. of mdpt
3. $\overline{HA} \parallel \overline{GB}$	3. Given
4. $\angle HAC \cong \angle BGC$	4. Alt. Int. $\angle$s Thm.
5. $\angle HCA \cong \angle BCG$	5. Vert. $\angle$s Thm.
6. $\triangle HAC \cong \triangle BGC$	6. ASA Steps 4, 2, 5

17.

Statements	Reasons
1. $\overline{WX} \perp \overline{XZ}$, $\overline{YZ} \perp \overline{ZX}$	1. Given
2. $\angle WXZ$, $\angle YZX$ are rt. $\angle$s.	2. Def. of $\perp$
3. $\triangle WXZ$, $\triangle YZX$ are rt. $\triangle$s.	3. Def. of rt. $\triangle$
4. $\overline{XZ} \cong \overline{ZX}$	4. Reflex. Prop. of $\cong$
5. $\overline{WZ} \cong \overline{YX}$	5. Given
6. $\triangle WZX \cong \triangle YXZ$	6. HL Steps 5, 4

18.

Statements	Reasons
1. $\angle S$, $\angle V$ are rt. $\angle$s.	1. Given
2. $\angle S \cong \angle V$	2. Rt. $\angle \cong$ Thm.
3. $RT = UW$	3. Given
4. $\overline{RT} \cong \overline{UW}$	4. Def. of $\cong$
5. $m\angle T = m\angle W$	5. Given
6. $\angle T \cong \angle W$	6. Def. of $\cong$
7. $\triangle RST \cong \triangle UVW$	7. AAS Steps 2, 6, 4

LESSON 4-6

19.

Statements	Reasons
1. M is mdpt. of $\overline{BD}$.	1. Given
2. $\overline{MB} \cong \overline{DM}$	2. Def. of mdpt.
3. $\overline{BC} \cong \overline{DC}$	3. Given
4. $\overline{CM} \cong \overline{CM}$	4. Reflex. Prop. of $\cong$
5. $\triangle CBM \cong \triangle CDM$	5. SSS Steps 2, 3, 4
6. $\angle 1 \cong \angle 2$	6. CPCTC

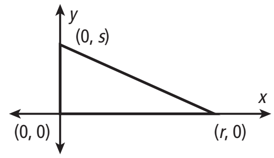
20.

Statements	Reasons
1. $\overline{PQ} \cong \overline{RQ}$	1. Given
2. $\overline{PS} \cong \overline{RS}$	2. Given
3. $\overline{QS} \cong \overline{QS}$	3. Reflex. Prop. of $\cong$
4. $\triangle PQS \cong \triangle RQS$	4. SSS Steps 1, 2, 3
5. $\angle PQS \cong \angle RQS$	5. CPCTC
6. $\overline{QS}$ bisects $\angle PQR$.	6. Def. of $\angle$ bisector

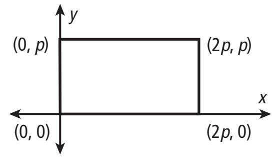
21.

Statements	Reasons
1. H is mdpt. of $\overline{GJ}$, L is mdpt. of $\overline{MK}$.	1. Given
2. $GH = JH$, $ML = KL$	2. Def. of mdpt.
3. $\overline{GH} \cong \overline{JH}$, $\overline{ML} \cong \overline{KL}$	3. Def. of $\cong$
4. $\overline{GJ} \cong \overline{KM}$	4. Given
5. $\overline{GH} \cong \overline{KL}$	5. Div. Prop. of $\cong$
6. $\overline{GM} \cong \overline{KJ}$, $\angle G \cong \angle K$	6. Given
7. $\triangle GMH \cong \triangle KJL$	7. ASA Steps 5, 6
8. $\angle GMH \cong \angle KJL$	8. CPCTC

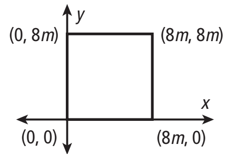
22. Check students' drawings; e.g., (0, 0), (r, 0), (0, s)



23. Check students' drawings; e.g., (0, 0), (2p, 0), (2p, p), (0, p)



24. Check students' drawings; e.g., (0, 0), (8m, 0), (8m, 8m), (0, 8m)



LESSON 4-7

25. Use coords. $A(0, 0)$, $B(2a, 0)$, $C(2a, 2b)$, and $D(0, 2b)$. Then by Mdpt. Formula, the mdpt. coords are $E(a, 0)$, $F(2a, b)$, $G(a, 2b)$, and $H(0, b)$. By Dist. Formula, $EF = \sqrt{(2a - a)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$, and $GH = \sqrt{(0 - a)^2 + (b - 2b)^2} = \sqrt{a^2 + b^2}$. So $\overline{EF} \cong \overline{GH}$ by def. of $\cong$.

26. Use coords. $P(0, 2b)$, $Q(0, 0)$, and $R(2a, 0)$. By Mdpt. Formula, mdpt. coords are $M(a, b)$. By Dist. Formula, $QM = \sqrt{(a - 0)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$, $PM = \sqrt{(a - 0)^2 + (b - 2b)^2} = \sqrt{a^2 + b^2}$, and $RM = \sqrt{(2a - a)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}$. So $QM = PM = RM$. By def., M is equidistant from vertices of $\triangle PQR$.

27. In a rt. $\triangle$, $a^2 + b^2 = c^2$.

$\sqrt{(3 - 3)^2 + (5 - 2)^2} = 3$,

$\sqrt{(3 - 2)^2 + (2 - 5)^2} = \sqrt{10}$,

$\sqrt{(2 - 3)^2 + (5 - 5)^2} = 1$, and $3^2 + 1^2 = \left(\sqrt{10}\right)^2$.

Since $9 + 1 = 10$, it is a rt. $\triangle$.

LESSON 4-8

28. Think: Use Equilat. $\triangle$ Thm. and $\triangle$ $\angle$ Sum Thm.

$$
\begin{aligned}
m\angle K = m\angle L &= m\angle M \\
m\angle K + m\angle L + m\angle M &= 180 \\
3m\angle M &= 180 \\
3(45 - 3x) &= 180 \\
-45 &= 9x \\
x &= -5
\end{aligned}
$$

29. Think: Use Conv. of Isosc. $\triangle$ Thm.

$\overline{RS} \cong \overline{RT}$

$RS = RT$

$1.5y = 2y - 4.5$

$4.5 = 0.5y$

$y = 9$

$RS = 1.5y = 13.5$

30. $\overline{AB} \cong \overline{BC}$
$AB = BC$
$x + 5 = 2x - 3$
$8 = x$
Perimeter $= AC + CD + AD$
$= 2AB + CD + CD$
$= 2(x + 5) + 2(2x + 6)$
$= 6x + 22$
$= 6(8) + 22 = 70$ units

CHAPTER TEST, PAGE 288

1. Rt. △
2. scalene △ ($AC = 4$ by Pythag. Thm)
3. isosc. △ ($AC = BC = 4$)
4. scalene △ ($BD = 4 + 3 = 7$)
5. $m\angle RTP = 2m\angle RTS$
$m\angle RTP + m\angle RTS = 180$
$3m\angle RTS = 180$
$m\angle RTS = 60°$
$m\angle RTS + m\angle R + m\angle S = 180$
$60 + m\angle R + 43 = 180$
$m\angle R = 77°$
6. $\overline{JL} \cong \overline{XZ}$
7. $\angle Y \cong \angle K$
8. $\angle L \cong \angle Z$
9. $\overline{YZ} \cong \overline{KL}$

10.

Statements	Reasons
1. T is mdpt. of $\overline{PR}$ and $\overline{SQ}$.	1. Given
2. $\overline{PT} \cong \overline{RT}$, $\overline{ST} \cong \overline{QT}$	2. Def. of mdpt.
3. $\angle PTS \cong \angle RTQ$	3. Vert. ∠s Thm.
4. $\triangle PTS \cong \triangle RTQ$	4. SAS Steps 2, 3

11.

Statements	Reasons
1. $\angle H \cong \angle K$	1. Given
2. $\overline{GJ}$ bisects $\angle HGK$.	2. Given
3. $\angle HGJ \cong \angle KGJ$	3. Def. of ∠ bisector
4. $\overline{JG} \cong \overline{JG}$	4. Reflex. Prop. of ≅
5. $\triangle HGJ \cong \triangle KGJ$	5. AAS Steps 1, 3, 4

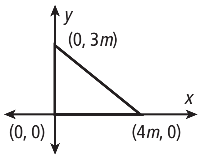
12.

Statements	Reasons
1. $\overline{AB} \perp \overline{AC}$, $\overline{DC} \perp \overline{DB}$	1. Given
2. $\angle BAC$, $\angle CDB$ are rt. ∠s.	2. Def. of ⊥
3. $\triangle ABC$ and $\triangle DCB$ are rt. △s.	3. Def. of rt. △
4. $\overline{AB} \cong \overline{DC}$	4. Given
5. $\overline{BC} \cong \overline{CB}$	5. Reflex. Prop. of ≅
6. $\triangle ABC \cong \triangle DCB$	6. HL Steps 5, 4

13.

Statements	Reasons
1. $\overline{PQ} \parallel \overline{SR}$	1. Given
2. $\angle QPR \cong \angle SRP$	2. Alt. Int. ∠s Thm.
3. $\angle S \cong \angle Q$	3. Given
4. $\overline{PR} \cong \overline{RP}$	4. Reflex. Prop. of ≅
5. $\triangle QPR \cong \triangle SRP$	5. AAS Steps 2, 3, 4
6. $\angle SPR \cong \angle QRP$	6. CPCTC
7. $\overline{PS} \parallel \overline{QR}$	7. Conv. of Alt. Int. ∠s Thm.

14.



15. Use coords. $A(0, 0)$, $B(a, 0)$, $C(a, a)$, and $D(0, a)$. By Dist. Formula,
$AC = \sqrt{(a - 0)^2 + (a - 0)^2} = a\sqrt{2}$, and
$BD = \sqrt{(0 - a)^2 + (a - 0)^2} = a\sqrt{2}$. Since
$AC = BD$, $\overline{AC} \cong \overline{BD}$ by def. of ≅.

16. Think: By Equilat. △ Thm., $m\angle F = m\angle G = m\angle H$.
$3m\angle G = 180$
$3(5 - 11y) = 180$
$5 - 11y = 60$
$-11y = 55$
$y = -5$

17. Think: Use △ ∠ Sum and Isosc. △ Thms.
$m\angle P + m\angle Q + m\angle PRQ = 180$
$2(56) + m\angle PRQ = 180$
$m\angle PRQ = 68°$
By Vert. ∠ and Isosc. △ Thms.,
$m\angle T = m\angle SRT = m\angle PRQ = 68°$.
Using △ ∠ Sum and Isosc. Thms.
$m\angle S + m\angle T + m\angle SRT = 180$
$m\angle S + 2(68) = 180$
$m\angle S = 44°$

18. It is given that $\triangle ABC$ is isosc. with coords. $A(2a, 0)$, $B(0, 2b)$, and $C(-2a, 0)$. D is mdpt. of $\overline{AC}$, and E is mdpt. pf $\overline{AB}$. By Mdpt. Formula, coords. of
D are $\left(\frac{-2a + 2a}{2}, 0\right) = (0, 0)$, and coords. of E are
$\left(\frac{2a + 0}{2}, \frac{0 + 2b}{2}\right) = (a, b)$. By Dist. Formula,
$AE = \sqrt{(a - 2a)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$, and
$DE = \sqrt{(a - 0)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$.
Therefore, $\overline{AE} \cong \overline{DE}$ and $\triangle AED$ is isosc.